HISTORICAL DYNAMIC MODELING (SIMULATION EXPERIENCE)

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ABSTRACT

The present work "synchronizes" the achievements in various fields of natural science (mathematics, cybernetics, information theory, sociology, economics and history) to understand the modern picture of the world. The conceptual model proposed in the paper is simplified and serves to illustrate certain socio-economic processes of the past and present. Undoubtedly, there is a "confirmation bias", since there is an element of subjectivism in every analysis. It can only be overcome by constructive criticism, not by ignoring facts, rejecting mathematical proofs and denying logical conclusions, for "Facts do not cease to exist because they are ignored" (Aldous Huxley).

KEYWORDS

History, social-economic model, mathematical simulation, optimization, development, strategy, equilibrium, non-stability, bifurcation, chaos, attractor, closed and open systems, system's theory, elite, society, community, sociology

Unlike the natural sciences, the socioeconomic field does not have many generally accepted facts that can nominate as rules. Only some of the fundamental laws of natural science can transpose as balance relationships. For example, the law of quantity transition interprete as the principle of the division of labor and, since the epoch of the industrial revolution and placed at the basis of the development of the world economy. The deepening of its division entails an increase in the risks of economic agents, which includes into the continuously increasing complexity of production chains. From a certain point, the absence of a mechanism to reduce such risks makes further deepening of the division of labor dangerous, due to which the socio-economic system finds itself in a state of deep crisis.

Mathematical description of the behavior of socio-economic subjects requires formalization of their principles of behavior. In its framework, not all behavior described, but the rational behavior associated with the adoption of a certain decision in a certain time. In such a problem, the principle of optimality looks as the list of rules, by which a subject determines its action, which best contributes to the achievement of the goal it is pursuing. Such a solution, satisfied the chosen principle, is called optimal, and their sequence in time is called a trajectory. The ultimate goal of model research is to find optimal trajectories for all subjects in progress.

The functioning of socio-economic systems is a self-learning process, transferring them from one evolutionary trajectory to another. The presence of several of them implies the possibility of choice. Thus, the mathematical model of socio-economical system is picture of transition from one state to another must contain the elements responding to instability without violating its stability. Practically, it realises through the establishment of feedback, which theoretically should smooth out the negative trends arising from the unpredictability of the impact of the external environment, and set the limits of intervention of the regulator, which recently has been increasingly often the state or mechanisms replacing and supplementing it.

David C. Wyld et al. (Eds): SIGV, AI & FL, SESBC, MLAEDU, DSCC, NLPTT, SCOM, SCM, CEEE - 2022
pp. 157-179, 2022. CS & IT - CSCP 2022 DOI: 10.5121/csit.2022.122213

No matter how the mathematical model built, there is always a problem of its adequacy to the described dynamic process. Modeling errors are of two types - initial data (by clarifying information and expanding the number of parameters) and selection (by varying approaches or changing the algorithm). In both cases, the criterion is the comparison of modeling results with the observed facts. If it is satisfactory, we can talk about adequacy. Formally, this looks as follows. Let the sequence $x_i(t)$ describes the behavior of the system, and $y_i(t)$ - its mathematical model. If for a given value $0 \leq \mathcal{E}$, the inequality

$| y_i(t) - x_i(t)| \leq \mathcal{E}$ for $\forall\, i \in N, 0 < t \leq \infty$ then the model is adequate to the process.

The main task of the regulator (control) of the system is to monitor and, if possible, fully or partially maintain the conditions under which the trajectory of a complex system maintains equilibrium. The simplest way of such type of influences is to create conditions and to form the environment for its self-organization. In this connection, it is possible to formulate seven basic principles of socio-economic strategy within the framework of the Theory of Catastrophes:

1. The change in the state of the socio-economic occurs by virtue of its internal mechanisms. External influences, although the cause of changes, but it never fully determines it.
2. The control defines as the transfer of an economic system from one state to another, which determinated by its objective constraints and the regulator's priorities. The preferred trajectory of movement is achieved by influencing the elements of the system, which forces it to evolve at the desired pace and in the desired direction.
3. The influence of influences on some elements of the system must be synchronized in time and space with the decisions made in its other parts.
1. The response of the system is directly proportional to the results of the control action only when the system is at the evolutionary stage of motion. The system, when at a bifurcation point, is subject to laws of a nonlinear nature. In this state, a resonant excitation leads to a greater effect than a stronger but unsynchronized excitation.
4. A resonance, albeit weak, leads to a greater effect than a strong, but uncoordinated influence on the system.
5. The main task of the control subsystem is to maintain (pseudo) stability of the system in the evolutionary phase and ensure transition to the best (in terms of the whole system) attractor during a disaster (In a situation of acute crisis, the possibility of a resonant control impact on the processes of self-organization is especially relevant).
6. The control subsystem must contain an element of self-regulation, which counteracts its uncontrolled proliferation, loss of competence or unproductive expenditure of resources.

Thus, to translate a social system into the right attractor, the regulator should be able to manage the chaos that its elements create. According to the concept of the German sociologist Ferdinand Tönnies [1], they are self-sufficient units that focus around two poles - community and society. “Gezellschaft” represents a rational relation based on material gain and thus lends itself to control by the state or/and the elite [2]. The "Gemeinschaft" is irrational because it determines by natural needs, which cannot always to formalize. The main problem in verifying the hierarchical model of human needs, or Maslow's pyramid, is that there is no reliable quantitative measure of satisfaction of human needs. Another weakness in the construction of the "pyramid of needs" is the definition of their sequence[3]. In addition to this, Maslow's theory cannot explain why some needs continue to be motivators even after they have been satisfied.

In the process of bifurcation, its influence intensifies and contributes to the increase in the amplitude of oscillations of individual elements and generates a large amount of noise. This problem posed in the field of natural sciences half a century ago and in some cases successfully solved. However, in the socio-economic field it has specific features. First, the nature of chaos must be clear. From a philosophical point of view, the haos are also a kind of order, or rather, in Hegel's understanding; chaos appears as a complex and logically unpredictable form of order [4]. In moral and ethical terms, chaos is associated with the absence of the usual order and external arbitrariness. Against the background of this emotional and literary stamp, the thesis of the creative, creative nature of chaos isn’t practically perceived by the public consciousness. At the same time, the constructive role of chaos is that it creates an environment that increases the degree of freedom of the elements of a complex system. Thanks to this, new ideas are generated, socio-economic and cultural innovations are born, and alternative ways of development emerge. In turn, expanding the range of possibilities and choices restores the old and creates new links between the elements of the emerging new system.

Thus, the development of the complex system is not, as in the generally accepted view, an alternation of order and chaos, but a combination of the two. At the same time, the order preserves only at the expense of chaos, which dampens the effects of the external environment. Thanks to it, the system becomes capable of responding adequately to the chaotic effects of the environment, and thereby maintains its stability. Thus, the creative role of chaos is not limited to the transition period, when it destroys non-viable elements of the old system. In addition, it performs certain protective functions, preventing an open system from reaching a state of "super-order". Chaos by itself is not capable of ensuring the self-organization of a complex system, and its presence does not guarantee a higher level of organization.

However, chaos not only generates order in various ways. Observed in many material processes, it describes strictly mathematically, i.e., it has a complex internal order [5]. In this connection, it makes sense to speak of the simplicity or complexity of the ordered structure. Due to the undeveloped criteria of the complexity of systems, we may speak of the possibility of observing and describing the order of chaos itself. Thus, chaos in a complex system is:

- The factor in the renewal of a complex organization and its adaptation to changes in the external environment;
- The way to synchronize the evolution of both internal subsystems and individual elements, and to prepare them for different variants of future development;
- The self-preservation mechanism;
- The trigger to select one of the attractors from the set of potentially possible ones. With a certain probability, the transition of a complex system to a new position can be realized without external intervention, but a more constructive approach is to identify the optimal conditions under which this probability will increase.

We can conclude that in a complex system the balance between the elements of order and chaos is changeable. Of course, from the point of view of control theory, it is preferable that the predominant part of the system elements follow order, but from the position of reliability theory, a certain degree of disorder is required. The task of managing chaos under these conditions is to try to preserve the stability of the system while searching for new alternatives for its development. Chaos in this situation acts as a tool, if not a restructuring, then an adjustment of the open system. By this reason, the nature of chaos precludes the possibility of controlling it. In practice, the instability of the trajectories of chaotic systems makes them more sensitive to control than other attractors and demonstrates better controllability and flexibility: the system is responsive to external influences and will slightly change its trajectory, preserving the momentum of motion. A successive series of small perturbations, applied at a specific place at a specific time, can strongly correct this process without taking the system out of the attractor. It is on this quality of chaos that the idea of its controlling includes: at the expense of one or a series of weak perturbations, change the trajectory of the system in a given direction, preserving its integrity.

Behind the simplicity of the concept lies a subtle and complex control mechanism. The success of it cannot guarantee. It not reduced to the set of rules and directives. The following should clearly understand:

- Any object cannot be controled in conditions of chaos:
- Perturbations cannot be strong enough not to destroy the impact element;
- The control should be sensitive to the state of the system (everything forms itself);
- It is necessary to strictly limiting the degree of freedom in the area of instability in order not to cause negative consequences,
- The integrity of the system must not be broken.

Consequently, the task of control a quasi-stable socio-economic system reduces to identifying the potential threats that exist on its trajectory of movement. It is necessary to highlight the elements that, in the global perspective, will become the foundations of a new structure. Therefore, the conclusion that the system must be unstable in order to reach a new level of development in the future does not seem so paradoxical. The application of the mathematical apparatus of catastrophe theory to the analysis of socio-economic models makes it possible to draw conclusions that largely contradict established stereotypes.

The concept of equilibrium underlies modern mathematical modeling of complex systems, including socio-economic systems. According to the "welfare theorems," a competitive equilibrium is the optimal state of a socio-economic system, and deviation from it is associated with a decrease in its efficiency. However, the description of the conditions of perfect competition is not constructive in the sense that it does not allow determining for a specific case whether these conditions preserve, and if not, and by how much they can deviate from the Walras's equilibrium solutions. In the case of imperfect competition, a generalization of the Bertrand model resolves the question of estimating deviation from the equilibrium condition. At the same time, Nash proved that a perfect NE exists for this type of finite extensive form game: at least one of the many equilibrium strategies would be played by hypothetical players having perfect knowledge of all 10 game trees [6].

The relationship of evolutionary theory and the concept of repeating games to these problems require clarification. The point here is the concept of "homo economicus", which plays a crucial role in economic theory, in particular in the model of economic equilibrium. According to this concept, the behavior of each individual (as well as some other elements of the socio-economic system) aims at maximizing his utility function from the consumption of various resources/benefits. Evolutionary game theory offers an alternative approach to describing the behavior of individual elements of a complex system, based on the models of natural selection, imitation and adaptation. This approach makes it possible to assess the limits of applicability of the above concept, to understand the possible mechanism of formation of the utility function.

To analyze socio-economic systems, it is appropriate to use the cybernetic notion of a viable system. This viable system model (VSM) describes any structure capable of maintaining its separate existence in a particular environment. One of the main features of viable systems is that they can adapt to changing environmental conditions, which allows the model to apply in the study of disasters. Cybernetics suggests that viable systems should view as recursive, i.e., some viable systems contain similar ones. This quality defined by the inventor of VSM as "cybernetic isomorphism"[7]. According to the recursive systems theorem, any viable system contains other viable systems, and is itself contained in a viable next-level system. In turn, this allows modeling of attractors, defining their structure, relationships and parameters on the example of one.

At the same time, loss of control can only occur due to low diversity (low intensity) of control, as Ashby thought when formulating the law [8]. It can describe that there is a state when the loss of control can take place at any high value of the control action, which reduces the entropy. The loss of control uniqueness occurs due to an increase in conditional entropy caused by the situation when the system parameters "behave" as independent random quantities, i.e. at the point of bifurcation.

Real dynamic processes are under the influence of many causes and disturbing influences (any mathematical formalization of the processes cannot fully consider). Therefore, the stage of identification of the system is the optimization stage of modeling, i.e. decision-making without regard to temporal factors. Based on the properties of the parameters (isomorphism and completeness) we can use the principle of convolution of the criteria. The evolutionary model of collective behavior based on the following considerations can qualitatively describe the struggle for resources:

1. There are n subjects (rivals) struggles for some resource (object) S. The total volume of the resource in each period of struggle bounds unchanging and equal to value S.
2. Each of the rivals i is a VSM. It generates a benefit g_i from acquiring/using a unit of a resource.
3. All rivals act rationally, i.e., they seek to maximize their benefits from own output.
4. Each opponent characterizes by a non-negative efficiency $0 < \alpha_i \leq 1$. This term refers to the socio-economic organization of the subject. It is a combined set of factors determining the level of consolidation of a particular socio-economic system in the face of external perturbations of various properties - threats and challenges and represents the top of Maslow's pyramid.
5. At each stage, all rivals seek to acquire such a share of the resourceq_i, from which they expect to get the maximum material benefit.
6. In the expectation of getting the maximum benefit at each stage of their struggle, the subjects incur costs $x_j \geq 0, j = 1, .. n$, which determine share in the distribution of the resource in a future session. At the same time, the expense cannot recover in any way and compensate from the income of the future;
7. If the rival has a negative yield, he drops out of the struggle and can never return again;
8. Rivals cannot cooperate with each other;
9. As the number of participants of struggle (rivals) changes, the parameters of the model change, and it transits to a new quality, i.e., a catastrophe occurs.

The volume of the received resource V_j and common benefit P_j of subject j during one stage (session) are:

$$V_j = S\frac{a_j x_j}{\sum_{i=1}^{n} a_i x_i}, \quad j = 1 \div n \quad \text{and} \quad P_j = V_j g_j - x_j = Sg_j \frac{a_j x_j}{\sum_{i=1}^{n} a_i x_i} - x_j, \; j = 1 \div n$$

This model is, in a sense, universal, since it corresponds to the definition of VSM. It can be used to analyze the competitive market segment, develop an election campaign strategy, predict the results of a military-political confrontation, etc. To adapt the model, it is only necessary to introduce universal units of measurement, set the appropriate resource parameters, correlate it with costs, and give an accurate definition of the level of efficiency. Based on the fact that each subject seeks to maximize its benefit at each stage of the rivalry, we obtain the following equation:

$$\frac{\partial P_j}{\partial x_j} = \frac{Sa_j g_j}{\sum_{i=1}^{n} a_i x_i}\left(1 - \frac{a_j x_j}{\sum_{i=1}^{n} a_i x_i}\right) - 1 = 0 \text{ or } a_j x_j^* = \frac{\sum_{i=1}^{n} a_i x_i^*}{Sa_j g_j}\left(Sa_j g_j - \sum_{i=1}^{n} a_i x_i^*\right) \quad (1)$$

As a rule, information about the opponent's actions is not available or is highly distorted. Therefore, at the next round of confrontation, each participant forms his own strategy based on previous experience, i.e. uses feedback. Thus, for participant *j* at iteration *m*, it looks:

$$a_j x_j(m) = \frac{\sum_{i=1}^{n} a_i x_i(m-1)}{Sa_j g_j}\left(Sa_j g_j - \sum_{i=1}^{n} a_i x_i(m-1)\right) \quad (2)$$

To simplify the analysis of behaviour during struggle, we introduce a summary characteristic of strategies of all rivals:

$$A_m = \sum_{i=1}^{n} a_i x_i(m) \quad (3)$$

The domain of this sum is the set of positive numbers ($A_m > 0$), because in the opposite case, rivalry simply does not arise. As a result, the iterative procedure (4) takes the form:

$$A_m = A_{m-1}\left(n - \frac{1}{S}\beta A_{m-1}\right) \quad (3^*)$$

This procedure is a positive feedback, since a change in the output signal of each of the rivals x_j leads to a change in the input signal of the others, which corrects the state of the system and speeds up the system's response to external changes. From the equation $A^* = A^*\left(n - \frac{1}{S}\beta A^*\right)$, we can find two equilibrium states of competing strategies: $A^* = 0$, and $A_1^* = \text{S}\frac{n-1}{\beta}$, where $\beta = \sum_{j=1}^{n} \frac{1}{\alpha_j g_j}$ – aggregate indicator of the ratio of efficiency and benefits, characterizing the spread of parameters. Its value fluctuates within

$$\frac{n}{a_1 g_1} \le \beta \le \frac{n}{a_n g_n} \quad (4)$$

In particular case, if $\frac{a_1 g_1}{a_n g_n} = 1$, than $\beta = \frac{n}{\sum_{i=1}^{n} a_i g_i} = \frac{1}{a_1 g_1}$. Taking into account a_j=1,$\forall j, j = 1 \ldots, n$, we have $= \frac{1}{g_i}$. It means that the generalized characteristic of the system is inversely proportional to the benefit of the most efficient opponent (rival). If we introduce the function of changing the cumulative strategies $y_m = A_m - A^*$, then struggle in the neighborhood of equilibrium A* described as:

$$y_m = y_{m-1}(2-n) - \frac{1}{S}\beta y_{m-1}^2 \ , \quad (5)$$

Since the presence of positive feedback, by definition, makes the system unstable, its most important characteristic is the stability of the equilibrium state. Based on the above considerations, from equation (7) it is possible to determine the necessary and sufficient condition for the asymptotic stability of the equilibrium A_1^*. It is determined by the inequality $|2-n|<1$, from which it follows that a stable resource division is achieved only in the case, when there are two rivals ($n=2$) acting rationally and are close enough to the equilibrum point. Thus, a stable division of the resource between two rivals always exists in the case of their rational behavior. This result is confirmed by practice in many areas of the socio-political life of various countries and peoples/ For example, the duopartial system, the split of world churches, the institute of co-government of nomadic empires, the division of the Roman and Frankish empires, Guelphs and Gibbelines, etc.

Based on the result obtained, we formulate a necessary and sufficient condition for the competitiveness of both rivals. From inequality $|y_m|<|y_{m-1}|$ we have, that $|\frac{1}{S}\alpha_n y_{m-1}|<1$. In case $n=2$, the equilibrium stability area is determined by the inequality $|y|<\frac{S}{\alpha_2}$. From (3) and (6) we receive $y_{m-1}=A_{m-1}-A^*, m\in N$ or $\left|A_{m-1}-\frac{S}{\alpha_2}\right|<\frac{S}{\alpha_2}$ and $0<A_{m-1}<\frac{2S}{\alpha_2}, \forall m\in N$. Local stability in the neighborhood of equilibrium corresponds to the inequality:

$$\left|\frac{y_m}{y_{m-1}}\right|<1 \Rightarrow \left|(2-n)-\frac{\beta}{S}y_{m-1}\right|<1, n\in N \quad \text{or} \quad S\frac{1-n}{\beta}<y_m<S\frac{3-n}{\beta}, m\in N. \tag{6}$$

The condition for the asymptotic stability of the equilibrium state is the condition: $\lim_{m\to\infty} y_m = 0$. It follows from this that the trivial state $A_0^*=0$ is unstable.

By equation (3*), we have $[A_m]=[A_{m-1}], \forall\, m, m \ni N$. It is not true, if $A_{m-1}\neq 0$.

In case *n=2* we have trajectories of the system around the point of trivial equilibrium. Fig. 1 illustrates the behavior of rivals striving for an indivisible or unique resource: winning a battlefield or an election in a single-mandate district. It follows from (6) that in the struggle for it at each particular stage the rivals do not lose only if there are two opponents and benefit only if there is one. In the case of four or more rivals some of them are guaranteed losses and are excluded from further procedure, which automatically leads to a change in the parameters of the model, i.e. a catastrophe.

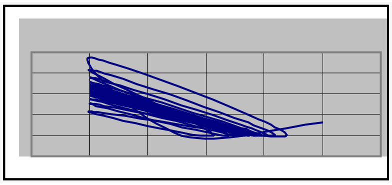

Fig 1. The struggle for an indivisible or unique resource

Awareness of the prospect of defeat may provoke one of the rivals to break rule |3|, i.e., to abandon rational behavior. In the latter case, he may believe that any strategy that ensures his success is justified by winning regardless of costs. However, he will be able to participate in further competition only if his costs do not exceed his winnings. Otherwise, today's winner, having no resources, according to condition |7| will not be able to participate in the competition from now on and, "dropping out" of the system will lead it to a disaster. Another variant of violation of the rules is collusion of several participants against the others. In this case, despite the fact that the creation of a coalition formally violates the established restrictions, it should be regarded as a single subject, the appearance of which only changes the parameters of the model. Thus, violation of rule |3| while obtaining a momentary benefit punishes the subject by excluding him from the system, in one case turning him into an outsider, and in the other depriving him of freedom of choice and turning him into an object of control, which is a part of a new, more complex system.

The struggle for a resource is futile for those rivals who do not have the necessary characteristics of efficiency and benefit. In some cases ($\alpha_j < \alpha_{i,} \quad g_j < g_i$) this is immediately obvious, but not in others ($\alpha_j \geq \alpha_{i,} \quad g_j \leq g_i$ Let us determine the conditions when participation in the struggle for a resource has a chance of success. Comparing the result (8) with the function of changing the cumulative strategies $y_m = A_m - A^*$, we find that its domain of definition exists only, when $n \leq 3$. From this we obtain two inequalities that determine the necessary condition for achieving stability around non-trivial point of equilibrium A_1^*:

$$\frac{2S}{\beta} < \sum_{i=1}^{n} a_j x_j(m) < \frac{2S(n-1)}{\beta}, m \in N$$

It should be noted that with an increase in the number of rivals, the area of their strategies expands linearly, creating new opportunities for struggle.

$$\frac{2S}{\beta} - \sum_{j=1,n,j\neq i}^{n} a_j x_j(m) \leq \frac{2S}{\beta} < a_i x_i(m) < \frac{2S(n-1)}{\beta} - \sum_{j=1}^{n} a_j x_j(m) \leq \frac{2S(n-1)}{\beta} - \sum_{j=1,i\neq i}^{n} a_j x_j(m)$$

Taking into account condition (4), we obtain the state A_1^* provided that $2 \leq n \leq 3$.

$$\frac{2S}{\beta} < \frac{2S(n-1)}{\beta}\left(1 - \frac{n-1}{\beta \alpha_i g_i}\right) < \frac{S(n-1)}{\beta} \text{ or } \frac{(n-1)^2}{(n-2)\beta} \geq \alpha_i g_i \geq 2\frac{n-1}{\beta} \quad (7)$$

Let's analyze the behavior of each of the rivals separately. Suppose that each of them has information about the actions of their opponents at the previous stage of struggle. In this case, the strategy $x_i(m)$ defines by expression:

$$x_i(m) = \frac{1}{a_i}\left(\sqrt{Sa_i g_i} - \sqrt{\sum_{j=1,j\neq i}^{n} a_j x_j(m-1)}\right)\sqrt{\sum_{j=1,j\neq i}^{n} a_j x_j(m-1)}, i = 1 \div n, m \in N$$

Condition (6) determines the profit limit for each of the rivals:

$$\frac{Sa_i g_i x_i(m)}{a_i x_i(m-1) + \sum_{j=1,j\neq i}^{n} a_j x_j(m-1)} - x_j(m) \geq 0, i = 1 \div n, m \in N$$

Hence, $0 \leq \sum_{j=1,j\neq i}^{n} a_j x_j(m-1) \leq Sa_j g_j, \forall i = 1 \div n, n \in N$ and $0 \leq x_i(m) \leq Sg_i, \forall i, i = 1 \div n, n \in N$.

Thus, the behavior of rivals, who always act rationally, is described by an iterative procedure:

$$x_i(m) = \begin{cases} \frac{1}{a_i}(\sqrt{Sa_i g_i} - \sqrt{\sum_{j=1,j\neq i}^{n} a_j x_j(m-1)})\sqrt{\sum_{j=1,j\neq i}^{n} a_j x_j(m-1)}, \\ when \sum_{j=1,j\neq i}^{n} a_j x_j(m-1) \leq Sa_i g_i, i = 1 \div n, m \in N \\ 0, when \sum_{j=1,j\neq i}^{n} a_j x_j(m-1) > Sa_i g_i, i = 1 \div n, m \in N \end{cases} \quad (8)$$

At the same time, the set $X^n = \{x_j : 0 \leq x_j(m) \leq Sg_j, j = 1 \div n, m \in N\}$ is invariant under procedure (8). In order to reach the equilibrium point, opponent i should use a strategy regardless of the actions of competitors:

$$x_i^* = \frac{1}{a_i}(\sqrt{Sa_i g_i} - \sqrt{\sum_{j=1,j\neq i}^{n} a_i x_j^*})\sqrt{\sum_{j=1,j\neq i}^{n} a_j x_j^*}, i = 1 \div n \quad (8^*)$$

$$\arg\max_{x_i} P(i) = \frac{1}{a(i)}(\sqrt{Sa_i g_i} - \sqrt{\sum_{j=1,j\neq i}^{n} a_j x_j^*})^2, i = 1 \div n$$

The equilibrium of strategies means that $x_i(m+1) = x_i(m), \forall i, m, i = 1 \div n, m \in N$. Solving the system of equations (8*), we obtain two equilibrium states:

$$B_0^* : x_i^* = 0, i = 1 \div n, \quad \text{and} \quad B_1^* : x_i^* = \frac{S(n-1)}{a_i} \frac{\sum_{k=1}^{n} \frac{1}{a_k g_k} - \frac{n-1}{a_i g_i}}{(\sum_{k=1}^{n} \frac{1}{a_k g_k})^2}, i = 1 \div n \quad (9)$$

Since $x_i(m) > 0, i = 1 \div n, m \in N$, the trivial equilibrium state is unstable. Therefore, the only acceptable strategy for rival *i* is B_1^*:

$$x_i^* = \frac{S(n-1)}{a_i} \frac{\beta - \frac{n-1}{a_i g_i}}{\beta^2} \quad (10)$$

Summing up the values at the equilibrium point for all rivals $i, i = 1 \div n$ we receive:

$$\sum_{i=1}^{n} a_i x_i^* = \sum_{i=1}^{n} \frac{S(n-1)}{\beta_n^2}(\beta - \frac{n-1}{a_i g_i}) = \frac{S(n-1)}{\beta_n^2}(n\beta - (n-1)\sum_{i=1}^{n} \frac{1}{a_i g_i}) = \frac{S(n-1)}{\beta} = A_1^*$$

Thus, the iterative procedures (3*) and (8*) have the only one and the same non-trivial equilibrium state of the resource division. In this case, the gain i in the state will be maximum and equal to:

$$P_i^* = Sg_i(1 - \frac{n-1}{\sum_{k=1}^{n} \frac{a_i g_i}{a_k g_k}})^2 = \frac{Sg_i}{\beta^2}(\beta - \frac{n-1}{a_i g_i})^2 \quad (11)$$

Agregate index β determine the relationship of efficiency a_i and benefit g_i. IT is a key characteristic of the behavior of the entire system. From (9) it follows that the equilibrium area depends by the number of rivals *n*. It is received from the inequality

$$n-1 \geq 2(n-2), \text{ or } n \in \{2,3\}.$$

In the case when three subjects compete, condition (9) takes the form of inequalities $a_i g_i > \frac{4}{\beta}, i \in \{1,2,3\}$. No one of them is not fulfilled.

At the same time, it defines the upper boundary at which struggle is preserved, i.e. the system remains open. Thus, if there are three or more competitors, the system will be in unstable equilibrium, i.e. it will remain open. It will permanently shake by "catastrophes" caused by "weaknesses" of some rivals, which will eliminate from the struggle, after each subsequent iteration, and the appearance of others. As the rivals'number changes, the system will experience "catastrophes" until there are only two left.

In the case of a duopoly/duumvirate, the behavior of both opponents will determine by the following equations:

$$x_1(m+1) = \frac{1}{a_1}\left(\sqrt{Sa_1g_1a_2x_2(m)} - a_2x_2(m)\right), \text{ if } Sa_1g_1 \geq a_2x_2(m)$$
$$x_2(m+1) = \frac{1}{a_2}\left(\sqrt{Sa_2g_2a_1x_1(m)} - a_1x_1(m)\right), \text{ if } Sa_2g_2 \geq a_1x_1(m)$$

A rivalry between them arises when $x_i(m) > 0, i = \{1,2\}, m \in N$. This means that the parameters of rivals must satisfy inequalities $0 \leq x_1(m) \leq S\frac{a_2g_2}{a_1}, m \in N$. Hence we conclude that struggle is possible only when $0 \leq x_1(m) \leq S\frac{a_2g_2}{a_1}, m \in N$ or $a_1g_1 - 4a_2g_2 \leq 0$. Hence we obtain the necessary and sufficient condition for the equilibrium of both strategies

$$1 \leq \frac{a_1g_1}{a_2g_2} \leq 4 \tag{12}$$

It can be argued that under the proposed model, the division of the resource between rivals is inevitable, because using strategies from the set X^2, they always reach an equilibrium point after a certain number of iterations. Fig. 2 illustrates this smooth transition of the system from one state to another. This does not mean that no ≪catastrophe≫ has occurred. It is simply that the parameters of the old equilibrium resource partition between the two rivals satisfy the necessary and sufficient conditions for the equilibrium state of the new one. In this case, a controlled process lends itself to a deterministic strategy.

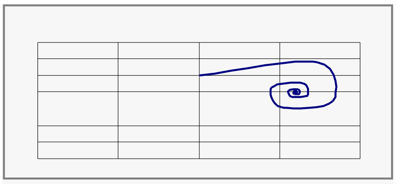

Fig 2. Equilibrium in a duopoly/diarchy

The specific value of the equilibrium state in the case of a duopoly/duumvirate can be found by solving the system of equations:

$$\begin{cases} a_1x_1 = \sqrt{Sa_1x_1a_2x_2} - a_2x_2 \\ a_2x_2 = \sqrt{Sa_2x_2a_1x_1} - a_1x_1 \end{cases} \quad (13)$$

To resolve the system (13), we receive two states of equilibrium $B_0 = (0;0)$ and $B_1 = \left(Sg_1 \frac{1}{(1+{}^{a_2g_2}/_{a_1g_1})^2}; Sg_2 \frac{({}^{a_2g_2}/_{a_1g_1})^2}{(1+{}^{a_2g2}/_{a_1g_1})^2} \right)$. From $\frac{{}^{a_2g_2}/_{a_1g_1}}{1+{}^{a_2g_2}/_{a_1g_1}} < 1$ we have $x_{1,B_1} < Sg_1$ и $x_{1,B_1} < Sg_2$, and $B_0 \in X^2, B_1 \in X^2$. We now define conditions for local asymptotic stability for a nontrivial equilibrium state. Its necessary and sufficient condition is the location of the eigenvalues of the Jacobi matrix in the equilibrium state of the iterative procedure (8*) inside the circle with radius 1. For the case $n = 2$, we get: $\frac{\partial f_1}{\partial x_1}\frac{\partial f_2}{\partial x_2} - \frac{\partial f_1}{\partial x_2}\frac{\partial f_2}{\partial x_1} \le 1$ $\frac{\partial f_1}{\partial x_1} = \frac{\partial f_2}{\partial x_2} = 0$. From (6) follows $\frac{\partial f_2}{\partial x_2} = \frac{a_1g_1 - a_2g_2}{2a_1g_2}$ and $\frac{\partial f_2}{\partial x_1} = \frac{a_2g_2 - a_1g_1}{2a_2g_2}$.

Hence we get that $\frac{(a_1g_1 - a_2g_2)^2}{4a_1g_1a_2g_2} \le 1$. Resolving this inequality, we obtain a necessary and sufficient condition for local stability around the equilibrium stage B_1:

$$3 - 2\sqrt{2} \le \frac{a_1g_1}{a_2g_2} \le 3 + 2\sqrt{2} \quad (14)$$

This result illustrates Fig. 3 and 4, in which competitors compete with parameters that satisfy the inequality $4 \le \frac{a_1g_1}{a_2g_2} \le 3 + 2\sqrt{2}$. Since the equilibrium is local, the system remains open, as illustrated by its phase trajectories.

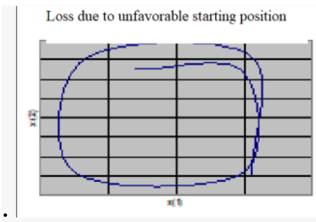

Fig 3. Loss to unfavorable position

On Fig.3 one of the rivals defeated by his opponent due to unfortunate initial conditions. Thus, there is a complete destruction of the system, which is out of equilibrium, and ceased to exist because even if the necessary conditions for stability observed. After it the system went to pieces and the only contender for the resource remained.

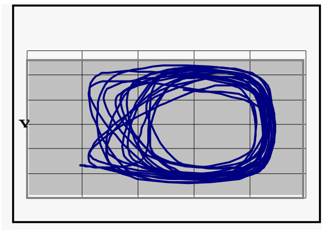

Fig 4. Rivals fail to reach parity due to different initial conditions

Fig.4 illustrates a slightly different situation: rivalry continues, but cannot reach equilibrium. It illustrates a situation where both competitors were at a disadvantage at the beginning of the process. Consequently, a cyclical process emerged, forcing both participants in the new system "to run by circle".

Thus, strategic planning will be effective only when the Nash equilibrium is stable and in some neighborhood around it. In the interaction of many participants, each pursuing their own goal (non-antagonistic conflict), the set of strategies chosen by them is called equilibrium, if a unilateral deviation of any of them from the chosen strategy leads to a decrease in its "gain. In all other cases it is multivariate even if there are two rivals whose parameters meet the necessary conditions for equilibrium stability. One way for each rival to win unconditionally is to change the benefit function, i.e. to deliberately destroy the system, and another way is to change the parameters α and g so as to drive the rival to bankruptcy. Such superiority of one of the rivals, i.e. the case when condition (14) is not satisfied, also leads to destruction of the system and establishment of monopoly.

For the general case in which there are *n* competitors for the resource *S*, an equilibrium state means that $x_i(m+1) = x_i(m), \forall i, m, i = 1 \div n, m \in N$. To resolve (9) we have two equilibrium states. Since $x_i(m) > 0, i = 1 \div n, m \in N$, the trivial equilibrium state is unstable. Hence, the only acceptable strategy for subject *i* is B_1^*. Analyzing the Jacobi matrix for n contenders, we obtain that the size of the area of local stability around the equilibrium state B_1^* is inversely proportional to the square of the number of contenders for the resource. A change in their number and the parameters ("benefitability" or efficiency) leads to a catastrophe of the system, which will change the aggregate index β and a new non-trivial equilibrium state, appears in point B_1^{**}.

The transition of the system from B_1^* to $B_1^{**}$is similar to the iterative procedure (8*) with new parameters and number of contenders. At a sufficient distance B_1^* with B_1^{**} (i.e. the location of the eigenvalues of the Jacobi matrix in the equilibrium state $B_1^{**}$of iteration procedure inside the circle of unit radius and B_1^* outside it), the catastrophe will continue. It will consist in the exclusion of new rivals whose characteristics do not meet the necessary and sufficient condition of local stability around the equilibrium B_1^{**}. As they are eliminated, it will change until the remaining *m* contenders *(m≤n, m, n* ∈ *N*) will not satisfy the local stability condition.

Since the beginning of the XVII century, the progress of the global world economy is based on the principle of the social division of labor. Its deepening inevitably entails an increase in the risks of the manufacturer, who integrated into ever more complex production chains. When innovation ceases to pay off, scientific and technological progress first slows down and then stops, after which the entire system enters a system of deep crisis, i.e. a catastrophe. The main problem of the development is the critical slowdown of scientific and technological progress and the decline the benefit of the use of the resource unit. These characteristics for subject *i* defined as efficiency α_i and specific yield g_i. Since each of the rivals can regulate the share of the resource consumed by him, in order to obtain the maximum benefit, from (13) it follows that his share will be equal to:

$$q_i = \sum_{k=1}^{n} q_k \frac{a_i x_i}{\sum_{j=1}^{n} a_j x_j} = Q\left(1 - \frac{n-1}{\beta a_i g_i}\right), \quad (15),$$

where $Q = \sum_{i=1}^{n} q_i$ - the common volume of used resource at the equilibrium B_1^*.

Let the connection between them be determined by the function that connects the system with the external environment Q(p) = φ (S_0, γ, p), where S_0 – maximal available resource, *p* – the benefit from the use of a resource unit (in economics – profit, in military – victory or loses of enemy), γ – a parameter relating its effectiveness to the volume actually available. In this case, for opponent *i* is defined as g_i= p-c_i, where c_i- system costs for the development of the resource in the segment Δt. In economics, this is the cost of the means of capital goods consumed during the production cycle and that part of the commodities that spent on the reproduction of labor resources, in informatics - entropy, in military affairs - equipment and movement of the armed forces.The benefit of *i* rival in non-trivial equilibrum point B_1^* defines as:

$$P_i(p) = Q(p) g_i(p)\left(1 - \frac{n-1}{\alpha_i g_i(p)\beta}\right)^2$$

It is possible to compose an iterative procedure that, on the time interval *Δt,* will correlate the amount of the resource with the other rivals, i.e. $q_i^m = q_i^{m-1} + \Delta \frac{\partial\varphi(q_i^{m-1})}{\delta p}$

For the case of diarchy/duopoly, from (13) we obtain the profit function for both rivals:

$$P_i(p) = \varphi(S_0, \gamma, p)\frac{a_i^2 g_i^3(p)}{(a_1 g_1(p) + a_2 g_2(p))^2}$$

Each of the rivals strives to maximize its benefit by influencing the level of benefit *p* through the share of the developed resource. Considering that φ (S_0, γ, p)=$S_0 e^{-\gamma p}$, we have:

$$\frac{\partial P_i}{\partial p} = \left(e^{-\gamma p}\frac{g_j^3}{(a_1 g_1 + a_2 g_2)^2}\right)' = -\gamma e^{-\gamma p}\frac{g_j^3}{(a_1 g_1 + a_2 g_2)^2} + \frac{3 g_j^2 (a_1 g_1 + a_2 g_2)^2 - 2(a_1 g_1 + a_2 g_2)(a_1 + a_2) g_j^3}{(a_1 g_1 + a_2 g_2)^4} e^{-\gamma p} = 0,\ j = 1,2$$

or $(3 - \gamma g_i)(a_1 g_1 + a_2 g_2) - 2(a_1 + a_2) g_i = 0,\ j = 1,2$

The set of solutions of this equation determines by the equations $g_2 - g_1 = 0$ and $a_1 g_1 + a_2 g_2 = 0$, from which it follows that $g_1 = g_2 = 0$. If $c_1 \neq c_2$ this condition is not true anytime, i.e. the system can exist only if there are two rivals who are absolutely identical in socio-economic terms. In this case, the solution of the problem, regardless of the initial conditions, is reduced to their parity|3|.

The situation fundamentally changes when $p_1 > p_2$(For example, rivals operate with different units of measure, for example, currencies). If both rivals try to maximize their benefit from resourse unit use *g(m)* by changing this characterstic. Appears iteration procedure:

$$\begin{cases} g_1(m) = g_1(m-1) + \Delta\left[(3-\gamma g_1(m-1))(a_1 g_1(m-1) + a_2 g_2(m-1)) - g_1(m-1)2(a_1+a_2)\right] \\ g_2(m) = g_2(m-1) + \Delta\left[(3-\gamma g_2(m-1))(a_1 g_1(m-1) + a_2 g_2(m-1)) - g_2(m-1)2(a_1+a_2)\right] \end{cases} \quad (16)$$

Since the eigenvalues of the Jacobi matrix for the iterative procedure (16) are negative, the only stable equilibrium for the system is:

$$\hat{p}_1 = c_1 + \frac{1}{\gamma}, \hat{p}_2 = c_2 + \frac{1}{\gamma} \quad (17)$$

From here, we can define the benefit function of each of the rivals as:

$$P_i(q) = Qg_i\left(1 - \frac{n-1}{\alpha_i g_i \beta}\right)^2 = p\frac{q_i^2}{Q} \quad (18)$$

For small values of γ, the function $Q = \frac{S_0}{1+\gamma p}$ is identical to its exponential representation $Q(p) = S_0 e^{-\gamma p}$. Thus, the "benefit's ability" p established depending on the total amount of the resource divided during one cycle by all rivals is equal to $p = \frac{1}{\gamma}(\frac{S_0}{Q} - 1)$, and its benefit function for subject *i* takes the form:

$$P_i(q) = \frac{q_i^2}{Q}\left(\frac{1}{\gamma}\left(\frac{S_0}{Q} - 1\right) - c_i\right) = \frac{S_0 q_i^2}{\gamma Q^2} - \frac{q_i^2}{Q}\left(\frac{1}{\gamma} + c_i\right)$$

and reaches its maximum, when

$$\frac{\partial P_i}{\partial q_i} = \frac{S_0 q_i}{\gamma Q^2}\left(2 - \frac{2q_i}{Q}\right) - \frac{q_i}{Q}\left(\frac{1}{\gamma} + c_i\right)\left(2 - \frac{q_i}{Q}\right) = 0$$

From here we obtain a system of nonlinear equations, from which we can find the equilibrium state in the distribution of the resource:

$$q_i = 2Q\frac{S_0 - Q - c_i\gamma Q}{2S_0 - Q - c_i\gamma Q}, i = 1, n$$

We can get an equation from which you can calculate the amount of resource used at the equilibrium point $\frac{2n-1}{2S_0} = \sum_{j=0}^{n} \frac{1}{2S_0 - (1+\gamma c_i)Q}$ (19)

and determine the value of the benefits of its unit at this point:

$$\sum_{i=1}^{n} \frac{p - c_i}{1 + 2\gamma p - \gamma c_i} = \frac{1}{2\gamma}$$

For rivals with the same costs, we get that the volume of the used resource and the benefitability of its unit will be equal to:

$Q = nq = 2S_0 \frac{n-1}{(2n-1)(1+c\gamma)}$, and $p = \frac{1}{\gamma}\left(\frac{S_0}{Q} - 1\right) = \frac{1}{\gamma}\left(\frac{(2n-1)(1+c\gamma)}{2(n-1)} - 1\right) > c$

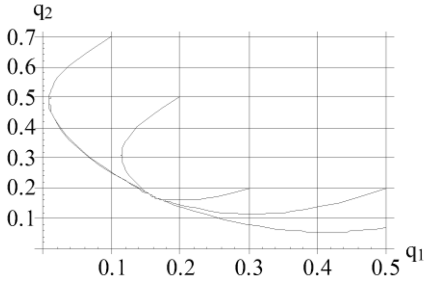

Fig 5. Division of a resource by two rivals with the equal costs

In the case $c_1 < c_2$ of the benefit of each of the rivals is:

$P_i(q) = \frac{S_0 q_i^2}{\gamma Q^2} - \frac{q_i^2}{Q}\left(\frac{1}{\gamma} + c_i\right)$, where $Q = q_1 + q_2$

Its maximum reaches, when

$\frac{\partial P_i}{\partial q_i} = \frac{S_0 q_i}{\gamma Q^2}\left(2 - \frac{2q_i}{Q}\right) - \frac{q_i}{Q}\left(\frac{1}{\gamma} + c_i\right)\left(2 - \frac{q_i}{Q}\right) = \frac{q_i}{\gamma Q_2}\left(2\frac{2S_0}{Q}(Q - q_i) - (1+\gamma c_i)(2Q - q_i)\right) = 0$ or

$\frac{2S_0}{Q}(Q - q_i) - (1+\gamma c_i)(2Q - q_i) = 0$

Thus, we get a system of equations describing the strategies of both competitors:

$$\begin{cases} 2\frac{S_0}{Q}q_2 - (1+\gamma c_1)(Q+q_2) = 0 \\ 2\frac{S_0}{Q}q_1 - (1+\gamma c_2)(Q+q_1) = 0 \end{cases} \quad (20)$$

The system of equations (20) has two roots corresponding to two equilibrium states. From (20) we obtain the points of the equilibrium division of the resource $E_1 = (\tilde{q}_1, \tilde{q}_2)$ and $E_1 = (\tilde{q}_1, \tilde{q}_2)$. In state $E_1 = (\tilde{q}_1, \tilde{q}_2)$ condition $p > c_2$ is always true. A similar result is obtained in the study of the iterative procedure.

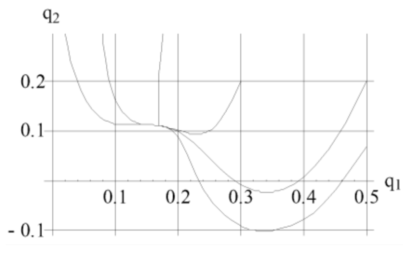

Fig 6. Division of resourse $c_1 > c_2$.

$$\begin{cases} q_1^m - q_1^{m-1} = \dfrac{\Delta}{\gamma}(2S_0 q_2^{m-1} - b_1(q_1^{m-1} + 2q_2^{m-1})(q_1^{m-1} + q_2^{m-1})) \\ q_1^m - q_2^{m-1} = \dfrac{\Delta}{\gamma}(2S_0 q_1^{m-1} - b_2(2q_1^{m-1} + 2q_1^{m-1})(q_1^{m-1} + q_2^{m-1})) \end{cases}$$

Analysis of the Jacobian matrix shows that what section of the resource at the point $E_1 = (\tilde{q}_1, \tilde{q}_2)$ is stable everything, but in $E_2 = (\bar{q}_1, \bar{q}_2)$ unstable for any values c_1 and c_2.

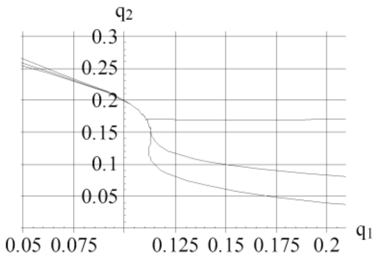

Fig 7. Division of resourse, $c_1 < c_2$.

If the number of competitors is extremely large ($n \to \infty$) benefitability from the use of a unit of resource will tend to the cost of its development at the worst of them, which will reduce the economic efficiency of the complex system as a whole. Thus, economic subjects drop out of the competition for the resource as their "assembly catastrophe" emerges and develops, i.e. internal problems accumulate. Hence, we can conclude that exoeconomic exploitation of the system's periphery increases the system's stability for some time and gives it an additional resource for transition to a new state. A positive example of the latter event is the Moscvia under the first Romanovs, the Austrian Empire in the early 19th century, Japan and Abyssinia at the turn of the 20th century and the Russian Federation at the turn of the 21st century.

It should be noted, that the above examples are exceptional cases. There are many more examples in history when socio-economic systems fail. The main reason for this is the policy of their regulator (the elite), who, for various reasons, is unable to adapt to new conditions. Typical examples are the Reformation and the collapse of the Habsburg Empire, the results of the clash of "eastern despotism" with the European powers, the collapse of the colonial system in the second half of the twentieth century and the sociopolitical degradation of several African countries in the last quarter of the twentieth century. Looking more closely at the situation, the aggressive external environment was only a catalyst for the decay of socio-political systems and initiated processes that destroyed their internal stability, and the cause was the loss of governance caused by the violation of the law of necessary diversity.

Since antiquity, the confrontation of the two sides for a resource has been quite rare: in their struggle, as a rule, there is a third force pursuing its own interests. During the Punic Wars, it was the Hellenistic world, during the Crusades - Byzantium, during the Anglo-French rivalry – Russian Empire. After the Napoleonic wars up to the middle of the twentieth century, the tripolar world became commonplace ("Holy Alliance"-Great Britain-independent America, Entente-Fourth Union-USA, Entente-Axis Countries-USSR), after which it became bipolar for a long time. Let us try to illustrate the reasons for this phenomenon with a model.

Let us continue the analysis of equation (19). Given that, we obtain a system of nonlinear equations that allow us to determine the benefitability of using the resource at the equilibrium point *p*. Under perfect competition, each competitor separately cannot influence it by changing the volume of the resource used. Accordingly, changing its volume, each of the rivals $i, i = 1,..,n, n \in N$ can consider their benefitability as unchanged and, correspondingly, has an incentive to extensive expansion as long as their costs are lower benefits $p \geq c_j$ $, j = 1, \dots. n, n \in N$.Since the costs of the rival j are equal to the costs of developing the resource c_j and the costs of fighting competitors x_j, we obtain the range of the function:

$$\frac{1}{\gamma}(\ln\frac{2n-1}{2n-2} - \ln(1 - \boldsymbol{\gamma c_1})) \leq \boldsymbol{p} \leq \frac{1}{\gamma}(\ln\frac{2n-1}{2n-2} - \ln(1 - \boldsymbol{\gamma c_n})), \qquad (22)$$

where characteristics are ranked in ascending order, i.e.. $c_1 < c_2, ., < c_n$ and $\gamma c_n < 1$.

To define the condition under which n rival will always participate in the resource section. For this, it is enough that the condition is fulfilled:

$$c_1 \leq c_n < p \leq \frac{1}{\gamma}(\ln\frac{2n-1}{2n-2} - \ln(1 - \boldsymbol{\gamma c_n})) \text{ , то есть } c_1 \leq c_n < \frac{1}{\gamma}(\ln\frac{2n-1}{2n-2} - \ln(1 - \boldsymbol{\gamma c_1}))$$

Solving this inequality with respect to n, we obtain the maximum number of rivals in the vicinity of the equilibrium of the system, determined by the values of the resource development costs of the best and worst of them:

$$n \leq 1 + \frac{1}{2((1-\gamma c_1)e^{\gamma c_n} - 1)} \qquad (23)$$

Due to its external nature, the parameter γ is not controllable. Its knowledge allows only to predict the system disaster, but not to change its outcome. At the same time, changes in the internal parameters of the system – used resource benefit g_i and efficiency of the system α_i is possible because it is associated with the self-organization of the system.

The "life" of the VSM system is shown in Fig.8. Three periods of rivalry for resource A, B and C can distinguished on it. Period A characterizes by a shortage of resources and contributes to an increase in the number of rivals, as a result of which there is a “chaos of free competition” caused by the growing volume of the developed resource. In period B, rivalry is gradually streamlined due to the exclusion of "unsuccessful" rivals, whose strategies, costs and efficiency do not correspond to the conditions of local equilibrium. At the same time, due to a random confluence of circumstances, subjects with the best performance indicators and the lowest costs may drop out of the struggle.

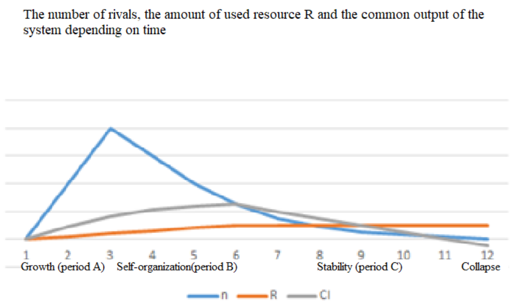

Fig 8 Stages of the existence of a VSM system

An area of fierce competition appears at stage C, where no more than four rivals (Religious wars, XIX century coalition wars, world wars, Cold War) dominate, which form an oligarchy with clearly fixed “rules of the game”. This follows from (7) and (17), whence we obtain the system of inequalities

$$\alpha_i \frac{1}{\gamma} > \frac{4}{\sum_{i=1}^{n} \gamma \frac{1}{\alpha_i}} \text{ or } \sum_{j=1, j\neq i}^{n} \frac{1}{\alpha_j} < \frac{3}{\alpha_i}, \quad \text{for } \forall i, i = 1, \dots, n$$

After sorting the characteristics in descending order $1 \geq \alpha_1 \geq \alpha_2 \geq \alpha_i \dots \geq \alpha_n \geq 0$, and remember, that $\frac{max}{i} \alpha_i = 1$, we have $\alpha_1 = 1$ and $\sum_{i=2}^{n} \frac{1}{\alpha_i} < 3$. Because $\frac{1}{\alpha_i} \geq 1$, we obtain the maximum number of rivals at the equilibrium point of the system, who have passed to stage C equal to four. In case n=4 rivals have equal characteristics, that is, they are indistinguishable from each other and represent a degenerate case, which can be seen as a case of forced synchronization At periods A and C, the rivals with the lowest costs receive the greatest gain, which due to this is less dependent on the efficiency of its use. The most difficult for them is the passage of the area of "proud competition", where, due to the growth in the number of rivals, it becomes necessary to fight for this indicator. After surviving phase B, they find themselves on an evolutionary trajectory and can reap the benefits of "balance", since the rivals who dropped out died and their remnants were absorbed. This situation continues either until the appearance of a rival with a lower cost of developing the resource, or a new product, or a change in the characteristics of rivals associated with a decrease in the benefit of the resource, the cost of its development and the efficiency of its use.

Even in the case of the formation of a “stable” oligarchy, one of the rivals will have the minimum cost of mastering a unit of the resource c_1 will set the optimal benefitability of the resource for itself, limiting the profit of its opponents, as illustrated by Fig. 9. For the worst of them, there comes a moment when the use of a resource at certain values of the yield of a unit of a resource gives zero effect and it leaves the system, while its opponents continue to operate in a changed external environment (with a reduced number of rivals). The most illustrative examples of such systems are environmental, associated with the depletion of habitats, resources and/or necessities, and the military.

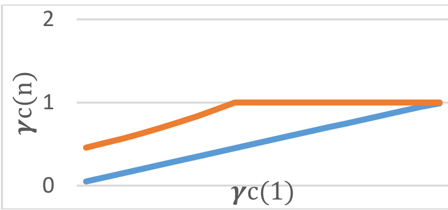

Fig 9. Resource yield functions for different rivals

In this case, the establishment of benefitability $p = \frac{1}{\gamma} + c_1$ rivalry will be reduced to determining the conditions of survival described above. Despite the fact that the “cheapest” rival will dictate its price, the struggle for the resource will be reduced to the competition of the “oligarchs” efficiency and will be determined by the conditions of survival (9), i.e. “tactical success guarantees nothing“[9].. This means that under certain conditions, even the “cheapest” opponent can be excluded from the competition.

An important consequence of this analysis is the possibility of mutual evaluation of rivals. Knowledge of equilibriums $B_1^*$and $B_1^{**}$for a particular rivals allows you to find out the conditions of his "catastrophe" and, therefore, to predict the prospects for the "survival" of a particular subject. Under these conditions, the task of the controller is to correct the individual parameters g and α of rivals in such a way that he remains part of the new system, taking the position of an ape "a wise monkey", which sits on a mountain and watches two tigers fight in the valley"(Mao Zedong).

Based on the results above, the following conclusion can be drawn, which, in principle, can be extended to all socio-economic processes.

1. If during catastrophe at least one of the elements of the old system satisfies the condition of sufficiency of equilibrium of the new one, then the system transitions to a new level of quality. At the same time, the elements of the system that satisfy the necessary conditions for the equilibrium of the new founded system will be preserved and integrated into the new system, while the rest will die (Due to cybernetic isoformism, the system will recover after some time).
2. If during the catastrophe none of the elements of the system satisfies the necessary conditions for the equilibrium of the new one, then it will self-destruct.
3. The presence of one or more elements of the system that satisfy the necessary condition for the existence of equilibrium of a new system generates a variety of unstable attractors.

The one of the possible methods for solving the chaos control problem is to determine the necessary conditions for the equilibrium of the system. Their presence makes it possible to determine those boundaries of influences on the trajectory of movement that will not allow the elements of the old system to self-destruct. Consequently, the approach to managing a socio-economic system using the apparatus of the theory of catastrophes is not to develop an algorithm for "ordering" chaos, but to determine the limiting boundaries between order and disorder at each stage of its development. This conclusion makes "the theory of controlled chaos" an attractive tool for the development of all sorts of social and political technologies, many of which vulgarize or pervert this concept.

A rival, whose resource development costs are obviously lower than any other competitor. will have a natural desire to get rid of all other competitors. By definition, it has the characteristics c_1 u $\alpha_{1.}$. Since his unit income will always be positive and, by definition, greater than the rest, he will strive to increase sales and capture the maximum resource δ_1.

$$\delta_1 = \frac{1}{\beta^2}(\beta - \frac{n-1}{\alpha_1(p-c_1)})^2 \ \rightarrow \ max \qquad (24)$$

This expression is non-negative for any values of значениях c_1 and $\alpha_{1.}$provided that the market is limited, i.e. is in stage C. According to rule |4|, the goal of the "Chinese" is to obtain maximum benefit/return, and, therefore, he will enter the struggle for the resource only on condition that in the vicinity of the equilibrium point $E_1(p_i,$ *i=1,.. n)* inequality $\beta - \frac{n-1}{\alpha_1(p-c_1)} > 0$ or $\alpha_1 > \frac{n-2}{\sum_{i=2}^{n}\frac{1}{\alpha_i}}$ is true.

It will always be able to get the maximum share of the resource when the function (24) reaches its maximum, i.e. $\frac{d\delta_1}{d\alpha_1} = 0$. The roots of this equation are the values

$$\beta = \frac{1}{\alpha_1(p-c_1)} \text{ и } \beta = \frac{n-1}{\alpha_1(p-c_1)} \text{ или } \sum_{i=2}^{n}\frac{1}{\alpha_i(p-c_i)} = 0 \text{ и } \sum_{i=2}^{n}\frac{1}{\alpha_i(p-c_i)} = \frac{n-2}{\alpha_1(p-c_1)} \qquad (25)$$

Equations (25) turn into identities when n=1, i.e. in the absence of rivals.

After establishing a monopoly on a resource, the system becomes quasi-closed, since there is always a theoretical possibility of the emergence of a new rival who will be able to master the part of the resource not covered by the monopoly. In his absence, the system continues to function unhindered in period C, representing a degenerate case. It is most consistent with victory in a total war with the complete annexation of the opponent's pocession (Punic Wars, the development of Siberia and the Wild West, the conquest of Americas and the Reconquista, the colonization of Australia, etc.). In order to prevent the emergence of a new rival, the regulator has to maintain the cost component associated with the retention of the resource. According to (3*), we obtain two values (the efficiency α_1 by definition becomes equal to 1): $x_1^0 = 0$ and $x_1^* = S_0 g_1$.

With the x_1^0 strategy, there is a complete "demobilization" of the rivalry apparatus, similar to the dissolution of the army, and there is a danger of the emergence of a spontaneous rival who can suddenly seize the resource and put the "winner" on the brink of disaster (the uprisings of slaves in the ancient world, peasants in the Middle Ages,"Black Death" epidemy). The x_1^* strategy is also not suitable, since it forces one to spend the entire available resource of the system on protection against ephemeral danger, which contradicts the principle of optimality. Therefore, following the instinct of self-preservation, the monopolist must apply any non-negative strategy that will ensure its survival in the event of a sudden invasion, and then allow it to realize its full power (landsknechts, 19th-century British army, US armed forces before World War I), i.e. $0 < x_1^0 \ll S_0 g_1$.

Every time the monopolist increases the volume of its product, it predetermines the potential decrease in its unit benefit. As a result, it will expand until its specific benefit from the use of the resource is equal to the costs of its development. Thus, it will always be higher than at the equilibrium point B_1^{**}, and the amount of resource used is lower. From a practical point of view, for such an equilibrium, the absolute indicators of benefits and costs do not matter - only their ratio is important (in the economic theory, the equilibrium of supply and demand can be at any level of production can always be achieved by changing the price). However, from the point of view of the internal structure of the socio-political system, it is not it that matters, but the internal structure of the elements of the system, their interaction each other and maintaining control over the resource. It can be shown that in the case of a monopolist strategy m holding the system at point B_1^{**} any new rival k whose parameters satisfy the inequality $\alpha_k g_k < \alpha_m g_m$ in competition for resource S will not receive a positive benefit, i.e. monopoly will be maintained. This situation seems to be an ideal case for solving the problem of the emergence of a rival: if the winner is able to fully master the available resource, then a closed system arises, which in the future awaits the "cusp catastrophe" [10].

However, quite often the opposite happens. for example, the impossibility of full development of the resources of the African and Asian colonies led to the emergence on their territory of European enclaves that exploited the periphery to one degree or another - trading posts, concessions and resettlement colonies. They can also include the creation of currency zones, industrial syndicates and technological platforms and refers to the case when the opponent's parameters satisfy the necessary and sufficient equilibrium condition. Its appearance actually breaks the monopoly and returns the system to zone B when there are several rivals. With a change in the cost of the resource, the emerging oligopoly/ oligarchy is again destroyed and the dominance of one of the rivals is restored, which will seek to "close" the system.

Preservation of the resource that is not exploited due to its high cost always provokes a recurrence of stage A, when new rivals appear in the "gray zone", mastering the unclaimed resource. In the case of overwhelming superiority of monopoly, they are not afraid and already at the next stage of competition they disappear or are integrated into the system as objects of control. The apparatus used by the system for struggle against them becomes its part, representing a "protective" subsystem, whose behaviour is described by the cost function $\mu(x)$. The main task of the "big brother" is to return the system to an equilibrium point B_1^*.

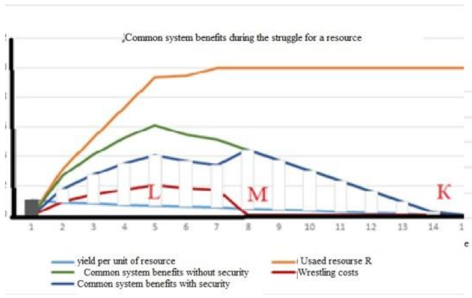

Fig 10. The struggle for a resource with the simultaneous appearance of rivals

Fig. 10 illustrates the effect of the function $\mu(x)$ on the common income of the superior system (α=1) under the conditions of entering the struggle for the resource from the first moment. At time L, the state of the environment stabilizes (i.e. the volume of the used resource reaches its possible maximum on the current technological platform) and a struggle for its redistribution between the strongest rivals begins, which lasts until the moment M, when victory achieved and the need for external protection eliminated until the system catastrophes at the time K.

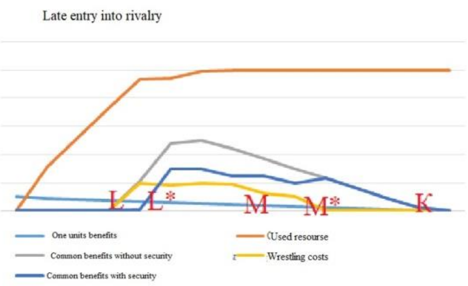

Fig 11. Late entry into the struggle for the resource

Fig. 11 illustrates the case where the strongest rival appeared at the time, when resource consumption reached its maximum. Consequently, it forced to incur substantially higher costs in the struggle for primacy (With some parameters and a certain strategy of the rivals, the system in Fig. 11 during the period L-L* could get losses and be displaced). A moment arose in the period when his yield was critically low but non-negative (Otherwise, he would have been forced to withdraw from the struggle for the resource). Since any socio-economic system consists of a number of related blocks, its highest efficiency of functioning achieves only in the case of an optimal ratio of interests by them. Decrease of output below the optimal level at least in one of them leads to imbalance and, as a consequence, the efficiency of the completely social system.

With the establishment of monopoly to keep it, a fundamentally different solution of the problem is restriction of the rival's access to the resource, i.e. to change the rules of behavior so that it cannot enter the area of local equilibrium. The simplest and most convenient way for regulator forced external synchronization, which repreents an "input payment" for a new rival. In the commerce case, it could be an import duty, a registration certificate or an insurance premium. For the socio-political model, these are medieval guilds and estates, electoral census, various forms of segregation, quotas on national representation, minimum threshold for passage to parliament, number of signatures for a referendum, etc.

Since the danger of sudden “invasion” continues to exist after achieving dominance, the winner was forced to maintain its defense in the period M - M*, which led to a decrease in benefits and reaches its greatest value later. In the process of achieving oligopoly, it begins to play the role of "big brother", carrying out the capture or absorption of weaker opponents. When achieving monopoly in the struggle for the resource, regulator transforms from “Gezellschaft” to “Gemeinschaft” [1] and takes over the functions of internal power [7]. This mechanism of the emergence of statehood, called the "theory of coercion", most thoroughly outlined in the works of R.L. Carneiro [11], who denies in his works the concept of the "Social Contract" [12], and other "voluntarity" theories.

Under the conditions of environmental constraints and land specificity, this phenomenon certainly takes place, but it is not absolute. Under certain conditions (Limited resources, aggressive external environment, ethnocultural homogeneity), proto-state can arise without military violence, as it happened, for example, in Iceland in the tenth century, the ancient polities and other compact in mathematical sense communities (technologically completed, territorially limited), having sufficient resources for self-development. In this case, there was mutual synchronization without the appearance of a "Big Brother” (by George Orwell). However, in most cases these systems turned out to close and due to the growth of entropy disintegrated or destroyed. Without denying the role of violence in the formation of the state, it should note that it largely related to expansion, i.e., the mastery of a resource available to a rival (the most successful rulers of antiquity were military leaders like Alexander the Great and Julius Caesar, not reformers like Lycurgus and Solon). In cases where it is impossible to absorb the entire resource due to various reasons, its mode of consumption transformed into an exopolitical economy based on plunder or its threat, known as polyudie. This socio-political phenomenon represents a symbiosis of "voluntarity" and "coercion" models of governance, representing the earliest "protective" system of the strongest of rivals, imposing external synchronization on the rest. The analogue of this phenomenon is military-political alliances, while mutual synchronization observed in political alliances.

When a monopoly is established, the tasks of the regulator, which previously exercised "external protection", first expand and then change in principle [7]. If at the beginning of the system's movement along the trajectory it represented a positive feedback, the task of which was to achieve the state of equilibrium B* and reflect the influence of the external environment, then from the moment of removal of all the competitors this connection changes its sign and becomes negative. Now its task is to keep the system in a state of dynamic equilibrium, when new competitors appear. Naturally, the methods of such control become completely different and require further research.

REFERENCES

[1] Tönnies' Complete Works (Ferdinand Tönnies Gesamtausgabe), 24 vols., critically edited by Lars Clausen, Alexander Deichsel, Cornelius Bickel, Rolf Fechner (until 2006), Carsten Schlüter-Knauer, and Uwe Carstens (2006–), Berlin/New York: Walter de Gruyter (1998–)

[2] Weber M. Wirtschaft und Gesellschaft, Tübingen, 1922.

[3] Maslow A. H. Motivation and Personality. — N. Y.: Harper and Row, 1954.

[4] Hegel’s Phenomenology of Spirit [Phänomenologie des Geistes*]*, translated by A.V. Miller, Oxford: Oxford University Press, 1977

[5] Lorenz, Edward Norton. "Deterministic nonperiodic flow". Journal of the Atmospheric Sciences. 20 (2): 130–141, 1963.

[6] Nash, John (1950) "Equilibrium points in n-person games" Proceedings of the National Academy of Sciences 36(1):48-49.

[7] Beer Stafford, Brain of the Firm; Allen Lane, The Penguin Press, London, 1972.

[8] Ashby, William Ross (1956). An Introduction to Cybernetics. London: Chapman & Hall.

[9] United States Army and United States Marine Corps. The U.S. Army/Marine Corps Counterinsurgency Field Manual. With Forewords by General David H. Petraeus and Lt. General James F. Amos and by Lt. Colonel John A. Nagl. With a New Introduction by Sarah Sewall, 2007. ISBN: 978-0-226-84151-9
[10] Thom, René. Structural Stability and Morphogenesis: An Outline of a General Theory of Models. Reading, MA: Addison-Wesley, 1989. ISBN 0-201-09419-3
[11] Carneiro, R L. Men and Cultures In Wallace, A F C (ed) Selected Papers of the Fifth International Congress of Anthropological and Ethnological Sciences (pp 229-234) Univ of Pennsylvania Press, Philadelphia, 1960.
[12] Jean-Jaques Russo, 'Du contrat social; ou Principes du droit politique. Amsterdam, 1762
[13] Kobishchanov Y.M. Polyudie: Phenomenon of Russian and World History. - M. 1995.

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