AUTONOMOUS VEHICLES LATERAL CONTROL UNDER VARIOUS SCENARIOS

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ABSTRACT

In this paper, the autonomous vehicle presented as a discrete-time Takagi-Sugeno fuzzy (T-S) model. We used the discrete-time T-S model since it is ready for the implementation unlike the continuous T-S fuzzy model. The main goal is to keep the autonomous vehicle in the centreline of the lane regardless the external disturbances. These disturbances are the wind force and the unknown curvature; they are applied to test if the autonomous vehicle moves from the centreline. To ensure that the autonomous vehicle remain on the centreline we propose a discrete-time fuzzy lateral controller called also steering controller.

KEYWORDS

Takagi-Sugeno model, Steering control, lane keeping, observers.

1. INTRODUCTION

Nowadays, vehicles have become an integral part of our daily lives. Most people worldwide need vehicles to move around, such as cars, buses, bicycles, taxis, etc. Also, those who are physically challenged use special vehicles. Due to the constant use of vehicles globally, so many people die every day because of vehicle collisions. Many studies declare that the main causes of these accidents are inattention, drowsiness, and illness [1] [2]. The purpose of autonomous cars lateral control is to maintain the vehicle in the lane under varied limits, and it is one of the most significant safety solutions. Over the last two decades, lateral stability control for autonomous vehicles has gotten a lot of attention from scientists and engineers, and numerous findings have been published [3]–[4]. Therefore, fruitful results with regard to stability and stabilization have been developed by researchers. By merging fuzzy logic and PID control, several authors propose a novel lateral control system design [5]. Others worked on controlling saturation through robust yaw control and stabilising lateral dynamics with concerns of parameter uncertainty. [6] A robust yaw-moment controller architecture for increasing vehicle handling and stability has been designed, taking into account parameter uncertainty and control saturation. However, the state vector only has two controllable parameters: the yaw rate and the sideslip angle; in this instance, there is insufficient information about the condition of the vehicles to allow for robust control. According to Sun et al. [7], the proposed proposal lacks a mathematical model, implying that no mathematical stability and stabilisation criteria exist to get access to system control. On the other hand, several researchers used deep learning and reinforcement learning approaches to examine vision-based autonomous driving [8]. Despite this, the model has no constraints, such as lateral wind force applied to the cars, unknown road curvature, or steering physical saturation. This means that the proposed solution is still far from being applicable in real-world driving conditions.Jiang and Astolfi [9] investigated an asymptotic stabilisation issue for a class of

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nonlinear under actuated systems. Its solution is used in the control of a vehicle's nonlinear lateral dynamics, together with back stepping and forward control design approaches. This approach demonstrated that a vehicle may track any conceivable reference at a constant speed using the established controller, and the lateral deviation converges to zero. The challenges in this scenario are that the longitudinal vehicle speed is constant, there is only one controllable internal variable, and the stability is local. Based on the aforementioned rationale, lateral control of autonomous cars requires a system that provides access to several vehicle states and swipes into a wide range of longitudinal speed. As a result, the Takagi–Sugeno (T–S) fuzzy models [10] have been widely acknowledged and used. The T-S is well-known as a valuable and popular paradigm for approximating complicated nonlinear systems. The nonlinear systems were approximated by fuzzy "blending" local linear models using a set of "IF-THEN" rules, which has piqued the control community's curiosity. In this paper, we will focus on discrete-time T-S fuzzy control. This controller is essentially based on feedback control, more specifically the parallel distributed control (PDC) law. In addition, the state vector will contain six internal variables to allow more accessibility for the autonomous vehicle control. In most cases, the state vector could be unreadable, noisy, or completely inaccessible. Based on this motivation, in our work we developed an observer called Luenberger multiobservers to ensure the reconstruction of the system state vector for accurate automatic steering control. This state vector will indeed be used in the control law equation for the fuzzy controller design. The stability and stabilization conditions will be based on the quadratic Lyapunov function. The Linear Matrix Inequality (LMI) approach is used in the optimization.

This paper is presented as follows. The vehicle modelling which is the part who is consecrated for the vehicle parameters and models. The third part presents the control design for the autonomous vehicle, which focuses on the stabilization of the autonomous vehicle. The final part is consecrated to show the results.

2. VEHICLE PARAMETERS AND MODELS

In this part, we show the different steps for the vehicle modelling. We start by introducing the vehicle parameters given in Table 1:

Table 1. Definition of parameters

Parameters	**Description**	**Value**
B_s	Steering system damping	5.73
C_f	Front cornering stiffness	57000 N/rad
C_r	Rear cornering stiffness	59000 N/rad
I_s	Steering system moment of inertia	0.02 kgm2
I_z	Vehicle yaw moment of inertia	2800 kgm2
K_p	Manual steering column coefficients	0.5
l_f	Distance from the CG to the front axle	1.3 m
l_r	Distance from the CG to the rear axle	1.6 m
l_s	Look-ahead distance	5 m
l_w	Distance from the CG to the impactcenter of the wind force	0.4 m
M	Mass of the vehicle	2025 kg

R_s	Steering gear ratio	16
σ_t	Tire length contact	0.13 m

2.1. Road-vehicle model

Using the well-known bicycle model described in [11]. The acquired model of the vehicle lateral dynamics is expressed as:

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \beta \\ r \end{bmatrix} \delta + \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} f_w. \quad (1)$$

Where β is the sideslip angle at the center of gravity (CG), and the yaw rate is r shown in Figure 1. In (1), the lateral wind force is fw, and the elements of the system matrices are written as follows:

$$a_{11} = -2\frac{(C_r + C_f)}{m\vartheta_x};\ a_{12} = 2\frac{(l_r C_r - l_f C_f)}{m\vartheta_x^2} - 1;$$
$$a_{21} = 2\frac{(l_r C_r - l_f C_f)}{I_z};\ a_{22} = -2\frac{(l_r^2 C_r + l_f^2 C_f)}{I_z \vartheta_x}.$$
$$b_1 = \frac{2C_f}{m\vartheta_x};\ b_2 = \frac{2 l_f C_f}{I_z};\ e_1 = \frac{1}{m\vartheta_x};\ e_2 = \frac{l_w}{I_z}.$$

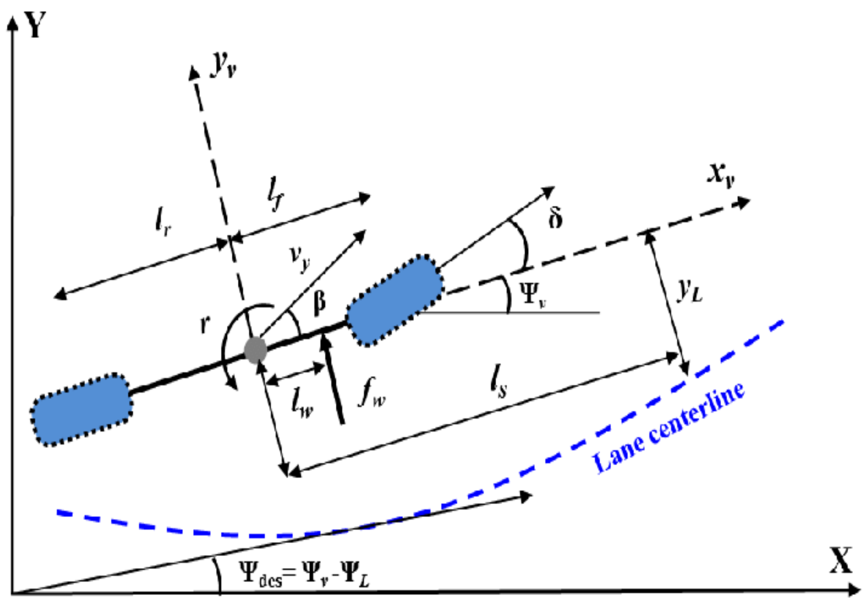

Figure 1. Lateral vehicle behavior modeling.

2.2. Vehicle position model on the road

The vehicle positioning dynamics on the road is described by [11]:

$$\begin{cases} \dot{y}_L = \vartheta_x \beta + l_s r + \vartheta_x \psi_L. \\ \dot{\psi}_L = r - \vartheta_x \rho_r. \end{cases} \quad (2)$$

Where y_L is the lateral deviation error from the centerline of the lane projected forward a look ahead distance l_s and ψ_L is the heading error between the tangent to the road and the vehicle orientation. The road curvature is indicated by ρ_r.

2.3. The vehicle steering model

The electronic power steering system is presented as [3]:

$$\ddot{\delta} = 2\frac{K_p C_f \sigma_t}{R_s^2 I_s}\beta + 2\frac{K_p C_f \sigma_t}{R_s^2 I_s}\frac{l_f}{\vartheta_x} r \cdots - 2\frac{K_p C_f \sigma_t}{R_s^2 I_s}\delta - \frac{B_s}{I_s}\dot{\delta} + \frac{1}{R_s I_s} T_s. \quad (3)$$

Where T_s is the steering torque, δ is the steering angle, l_s is the inertia moment of the steering column, B_s is the damping factor of the column, R_s is the reduction ratio of the column, σ_t is the width of the tire contact finally, the manual steering column coefficient is K_p.

2.4. The autonomous vehicle model

Based on (1), (2) and (3) the autonomous vehicles model is:

$$\dot{x}(t) = A_v x(t) + B_{vu} u(t) + B_{vw} w(t). \quad (4)$$

Where $x = \left[\beta r \psi_L y_L \delta \dot{\delta}\right]^T$ is the vehicle vector state, $w = [f_w \rho_r]^T$ is the disturbance vector, and $\mathrm{u} = \mathrm{T_s}$ is the input vector. The control-based system matrices in (4) are expressed as:

$$A_v = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 & b_1 & 0 \\ a_{21} & a_{22} & 0 & 0 & b_2 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ v_x & l_s & v_x & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ a_{61} & a_{62} & 0 & 0 & a_{65} & a_{66} \end{bmatrix}; B_{vlw} = \begin{bmatrix} e_1 & 0 \\ e_2 & 0 \\ 0 & -v_x \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}. \mathrm{B_v} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\mathrm{R_s I_s}} \end{bmatrix}.$$

We have

$$a_{61} = 2\frac{K_p C_f \sigma_t}{R_s^2 I_s},\ a_{62} = 2\frac{K_p C_f \sigma_t}{R_s^2 I_s}\frac{l_f}{v_x},\ a_{65} = -2\frac{K_p C_f \sigma_t}{R_s^2 I_s},\ a_{66} = -\frac{B_s}{I_s}.$$

As a result, the control analysis and development in this work will be based on the discrete-time system described below (5).

$$\mathrm{x(k+1) = A\,x(k) + B_d\,u(k) + B_{lw} w(k)} \quad (5)$$

$\mathrm{Te} = 0.01$ second is the discretization time. A is the discrete-time matrix of $\mathrm{A_v}$, $\mathrm{B_d}$ is the discrete-time matrix of $\mathrm{B_v}$ and $\mathrm{B_{lw}}$ is the discrete-time matrix of B_{vlw}.

3. AUTONOMOUS VEHICLE STABILIZATION ARCHITECTURE

In this section, we present the control design steps for the aim purpose is to control the autonomous vehicle presented in (5).

3.1. The autonomous vehicle T-S model

The discrete-time (T-S) system is presented by fuzzy IF-THEN rules.

The Rule i is of the form:

IF $E_1(k)$ is Q_{1i} and ... $E_n(k)$is Q_{ni} THEN

$$\begin{cases} x(k+1) = A_i x(k) + B_i u(k). \\ y(k) = C_i x(k). \end{cases} \quad (6)$$

Where $E_1, \ldots, E_n$ are linear functions, $Q_{1i}, \ldots . Q_{ni}$ are the fuzzy sets, and n is the number of the rules. In addition, $x_k \in \mathbb{R}^n$ is the state vector; $u_k \in \mathbb{R}^p$ is the measurable output vector; A_i, B_i and C_i are the system matrices with appropriate dimension, In addition, the premise variables are represented by the vector $\Gamma(k) = [\Gamma_1(k) \ldots . \quad \Gamma_q(k)]$. The autonomous vehicle model is:

$$x(k+1) = \sum_{i=1}^{n} \omega_i \left(\Gamma(k)\right)\left(A_i x(k) + B_i u(k) + B_i^w w(k)\right). \quad (7)$$

$$\begin{cases} \sum_{i=1}^{n} \omega_i \left(\Gamma(k)\right) = 1 \\ \omega_i(\Gamma(k)) \geq 0. \end{cases} \quad (8)$$

With

$$\omega_i(\Gamma) = \frac{w_i(\Gamma(t))}{\sum_{i=1}^{n} w_i(\Gamma(t))} \; ; \; w_i\left(\Gamma(t)\right) = \prod_{j=1}^{q} M_{ji}(\Gamma(t)).$$

The proposed Lyapunov function is:

$$V\left(x(k)\right) = x_k^T S \, x_k. \quad (9)$$

The discrete-time T-S fuzzy system described in equation (7) is asymptotically stable if there exists a common symmetric matrix $S = S^T > 0$ such that the following LMI are feasible [12] [13]:

$$\begin{cases} S > 0. \\ A_i^T S \, A_i - S < 0. \end{cases} \quad (10)$$

3.2. Autonomous vehicle stabilization

The control law is:

$$u(k) = -\sum_{i=1}^{n} \omega_i\left(\Gamma(k)\right)[G_i x(k)]. \quad (11)$$

Using the discrete-time T-S system previously described in (7), a closed loop control given by the new PDC law:

$$\begin{cases} x(k+1) = \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i\left(\Gamma(k)\right)\omega_j\left(\Gamma(k)\right)\vartheta_{ij} x(k). \\ \vartheta_{ij} = \left(A_i - B_i G_j \right). \end{cases} \quad (12)$$

Hypothesis 1: The T-S system is locally controllable as described in [14].

The discrete time T-S models stabilization conditions for a closed-loop PDC controller are that there exists a symmetric matrix $S > 0$ as well as gains $G_i, \forall i \in I_n$ satisfying:

$$\begin{cases} C_{discrete}(\vartheta_{ii}, S) < 0, \forall i \in I_n, \\ C_{discrete}(\vartheta_{ij}, S) \leq 0, \forall i, j \in I_n^2 \\ \quad \omega_i(\Gamma(k))\omega_j(\Gamma(k)) \neq 0. \end{cases} \tag{13}$$

And,

$$C_{discrete}(\vartheta_{ij}, S) = \left[\frac{\vartheta_{ij}+\vartheta_{ji}}{2}\right]^T S \left[\frac{\vartheta_{ij}+\vartheta_{ji}}{2}\right] - S. \tag{14}$$

The conditions are:

$$(A_i - B_i G_i)^T S (A_i - B_i G_i) - S < 0. \tag{15}$$

Multiplying (15) in pre and post by S^{-1}, we come by the following inequality:

$$S^{-1}(A_i S^{-1} - B_i G_i S^{-1})^T S (A_i S^{-1} - B_i G_i S^{-1}) > 0. \tag{16}$$

It's supposed that $X = S^{-1}$ and $H_i = G_i S^{-1}$, thus we get the following:

$$X(A_i X - B_i H_i)^T S\,(A_i X - B_i H_i) > 0. \tag{17}$$

The inequality (17) can be presented in LMI form by the application of the Schur complement as follows:

$$\begin{bmatrix} X & * \\ A_i X - B_i H_i & X \end{bmatrix} > 0 \;\forall\, i \in I_n. \tag{18}$$

By application of the same approach and the same steps, the condition $C_{discrete}(\vartheta_{ij}, S) \leq 0$ is given by:

$$\begin{bmatrix} X & * \\ \frac{A_i + A_j}{2} X - \frac{1}{2}(B_j H_i + B_i H_j) & X \end{bmatrix} \geq 0 \;\forall (i, j) \in I_n^2, i < j. \tag{19}$$

The discrete-time T-S system described in equation (7) are globally asymptotically stable via the novel PDC control law, if there are symmetric matrices such as $S = S^T > 0$ and $M = M^T \geq 0$ which verifies [15]:

$$\begin{cases} C_{discrete}(\vartheta_{ii}, S) + (r-1)M < 0, \forall i \in I_n, \\ C_{discrete}(\vartheta_{ij}, S) - M \leq 0, \forall i, j \in I_n^2, i < j, \\ \quad \omega_i(\Gamma(k))\omega_j(\Gamma(k)) \neq 0. \end{cases} \tag{20}$$

$$C_{discrete}(\vartheta_{ij}, S) = \left[\frac{\vartheta_{ij}+\vartheta_{ji}}{2}\right]^T S \left[\frac{\vartheta_{ij}+\vartheta_{ji}}{2}\right] - S. \tag{21}$$

Using theorem announced in [13] helps to decrease the conservatism. **Theorem 1:** if there exist matrices $S = S^T > 0$, $M_{ij} = M_{ij}^T$ and matrices G_i which verifies:

$$\begin{cases} C_{discrete}(\vartheta_{ii}, S) + (r-1)M < 0, \forall i \in I_n. \\ C_{discrete}(\vartheta_{ij}, S) - M \leq 0, \forall i, j \in I_n^2, i < j. \\ \omega_i(\Gamma(k))\omega_j(\Gamma(k)) \neq 0. \end{cases} \quad (22)$$

$$M = \begin{bmatrix} M_{11} \ldots. & M_{1n} \\ M_{1n} \ldots. & M_{nn} \end{bmatrix}. \quad (23)$$

Applying the previous equations, the autonomous vehicle model presented in (7) is globally asymptotically stable.

$$\begin{cases} X = S^{-1}. \\ Y_{ii} = XM_{ij}X. \\ G_i = H_i X^{-1}. \\ \forall i \in I_n. \end{cases} \quad (24)$$

3.3. Multiobservers design

To perform the control of the autonomous vehicle we need the entire state vector $x(k)$. To achieve this goal we applied the following observer's equation:

$$\begin{cases} \widehat{X}(k+1) = \sum_{i=1}^{n} \omega_i(\Gamma(k))\left(\left(A_i\hat{x}(k) + B_i u(k)\right) + N_i\left(y(k) - \hat{y}(k)\right)\right). \\ \hat{y}(k) = \sum_{i=1}^{n} \omega_i\left(\Gamma(k)\right)C_i\hat{x}(k). \end{cases} \quad (25)$$

The error state vector is written as:

$$\widetilde{x}(k) = x(k) - \hat{x}(k). \quad (26)$$

Knowing that, the dynamics of the state vector error is given by:

$$\begin{cases} \hat{x}(k+1) = \sum_{i=1}^{n}\sum_{j=1}^{n} \omega_i\left(\Gamma(k)\right)\omega_j\left(\Gamma(k)\right)\alpha_{ij}\tilde{x}(k). \\ \alpha_{ij} = A_i - H_i C_j, \forall (i,j) \in I_n^2. \end{cases} \quad (27)$$

In fact, the design of Luenberger observers requests the computing of local gains $N_i \in I_n$ to guarantee the convergence to 0 of the state vector error dynamics. In addition, we should guarantee that $S = S^T > 0$ and matrices $N_i \in I_n$ valid the following conditions:

$$\begin{cases} C_{discrete}(\alpha_{ii}, S) < 0, \forall i \in I_n. \\ C_{discrete}(\alpha_{ij}, S) \leq 0, \forall i, j \in I_n^2. \\ \omega_i(\Gamma(k))\omega_j(\Gamma(k)) \neq 0. \end{cases} \quad (28)$$

$$C_{discrete}(\alpha_{ij}, S) = \left[\frac{\alpha_{ij} + \alpha_{ji}}{2}\right]^T S \left[\frac{\alpha_{ij} + \alpha_{ji}}{2}\right] - S. \quad (29)$$

The equations (28) and (29) can be written as LMIs applying the Schur complement:

$$\begin{bmatrix} S & * \\ S\frac{A_i+A_j}{2} - \frac{1}{2}\left(H_jC_j + H_jC_i\right) & S \end{bmatrix} \geq 0 \quad (30)$$

When $i < j$.

We improve the observers by using the following theorem. **Theorem 2:** the multiple Luenberger observers are globally asymptotically stable if there exist symmetric matrices $S > 0$, M_{ij} and $N_i \in I_n$ that satisfy:

$$\begin{cases} C_{discrete}(\alpha_{ii}, S) + M_{ii} < 0, \forall i \in I_n. \\ C_{discrete}\left(\alpha_{ij}, S\right) + M_{ij} \leq 0, \forall i, j \in I_n^2. \\ \begin{bmatrix} M_{11} & M_{1n} \\ M_{1n} & M_{nn} \end{bmatrix}. \\ \omega_i\left(\Gamma(k)\right)\omega_j\left(\Gamma(k)\right) \neq 0. \\ \alpha_{ij} = A_i - N_iC_j, \forall(i,j) \in I_n^2. \end{cases} \quad (31)$$

The LMI's are given by:

$$\begin{cases} S > 0. \\ \begin{bmatrix} S - M_{ii} & * \\ SA_i - H_iC_i & S \end{bmatrix} > 0 \ \forall\, i \in I_n. \\ \begin{bmatrix} S - M_{ij} & * \\ S\frac{A_i+A_j}{2} - \frac{1}{2}(H_iC_j + H_jC_i) & S \end{bmatrix} \geq 0 \ \forall\, i < j. i, j \in I_n^2. \\ M = \begin{bmatrix} M_{11} \dots. & M_{1n} \\ M_{1n} \dots. & M_{nn} \end{bmatrix}. \\ H_i = SN_i. \end{cases} \quad (32)$$

4. RESULTS

We applied the enhanced PDC control law design called classic PDC [16] to the discrete-Time T-S fuzzy model representing the autonomous vehicle system to ensure the lateral control purpose under certain constraints. Equations (25) and (32) give:

$$\begin{cases} S = X^{-1}. \\ G_i = H_iS. (33) \\ H_i = SN_i. \end{cases}$$

Based on (33), we get the following gains and matrices:

$$S = e^{-03}\begin{bmatrix} 0.0513 & 0.0139 & 0.0693 & 0.0120 & 0.1275 & 0.0070 \\ 0.0139 & 0.0062 & 0.0235 & 0.0041 & 0.0400 & 0.0025 \\ 0.0693 & 0.0235 & 0.1267 & 0.0196 & 0.21158 & 0.0121 \\ 0.0120 & 0.0041 & 0.0196 & 0.0051 & 0.0368 & 0.0019 \\ 0.1275 & 0.0400 & 0.2158 & 0.0368 & 0.4979 & 0.0266 \\ 0.0070 & 0.0025 & 0.0121 & 0.0019 & 0.0266 & 0.0029 \end{bmatrix}$$

$$G_1 = [\,538.7210 \quad 88.1555 \quad 137.5593 \quad 21.1442 \ - 146.3325 \ \ - 56.7097].$$

$$G_2 = [545.2576 \quad 58.9360 \quad 139.5243 \quad 21.1255 \ - 148.8607 \ \ - 56.7512].$$

For multi-observer gains are:

$$\mathrm{N1} = e^{03}\begin{bmatrix} 0.0116 & 0.0007 \\ 0.0828 & -0.0020 \\ 0.0285 & -0.0003 \\ 0.2673 & -0.0020 \\ -0.0196 & -0.0273 \\ 9.3447 & 7.3696 \end{bmatrix}, \mathrm{N2} = e^{04}\begin{bmatrix} 0.0026 & 0.0000 \\ 0.0152 & -0.0001 \\ 0.0076 & -0.0000 \\ 0.0624 & -0.0001 \\ 0.0074 & -0.0023 \\ -1.2496 & 0.6264 \end{bmatrix}.$$

We tested our controller with different scenarios. The first scenario is to assume that the autonomous vehicle system will start far from the origin with different orientation. The initial state vector $x_0 = [0; 0.02; 0.04; 0; 0; 0.9]$, which is not the system equilibrium point [0;0;0;0;0;0], example, the lane centreline. We can certainly observe in figure 4-8 that our stabilization control law converges all the state variables to zero, which means that our autonomous vehicle reaches the centreline of the lane. These results prove the robustness of our controller. The second scenario is to apply a disturbance mitigation. We subjected the autonomous vehicle to a lateral wind force of 1500 Newton. Figure 2 and Figure 3 show a remarkable robust stabilization. Our model showed robustness and effectiveness against the disturbances.

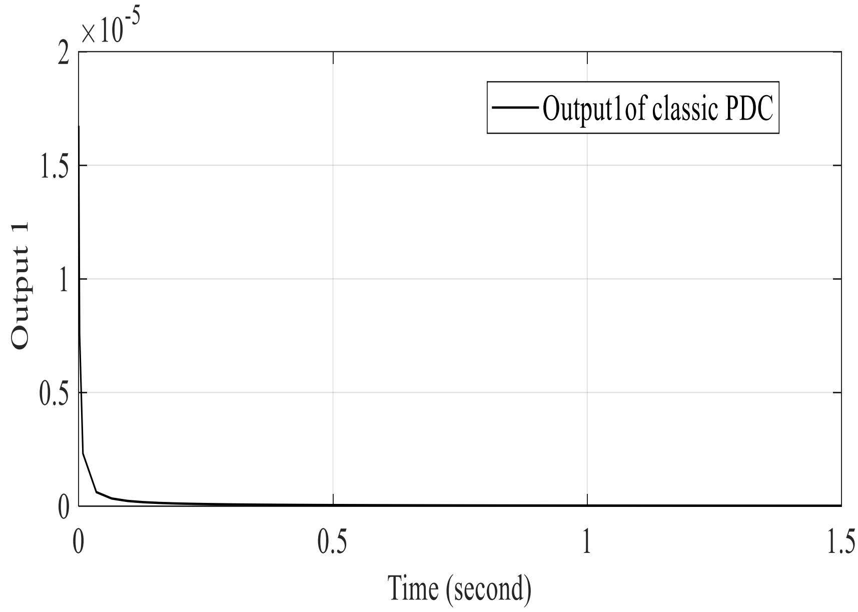

Figure 2. Stabilized output1

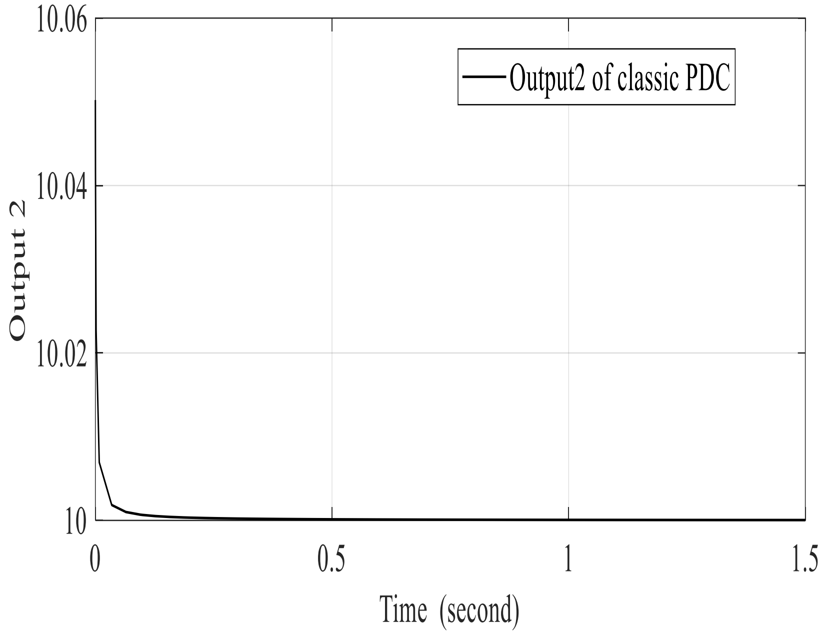

Figure 3. Stabilized output2.

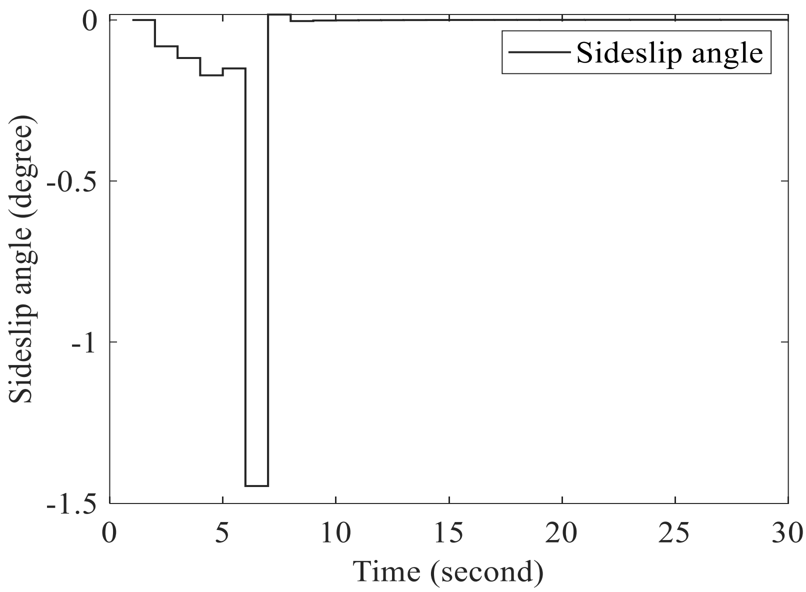

Figure 4. Sideslip angle.

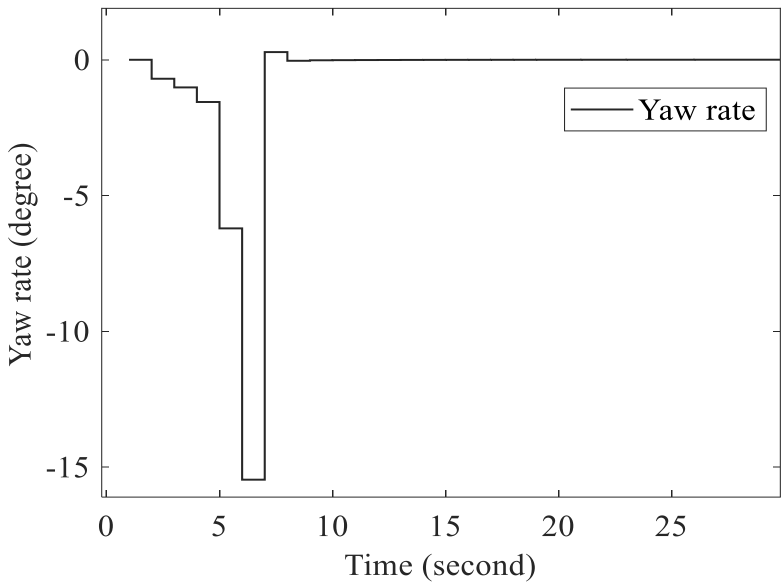

Figure 5. Yaw rate.

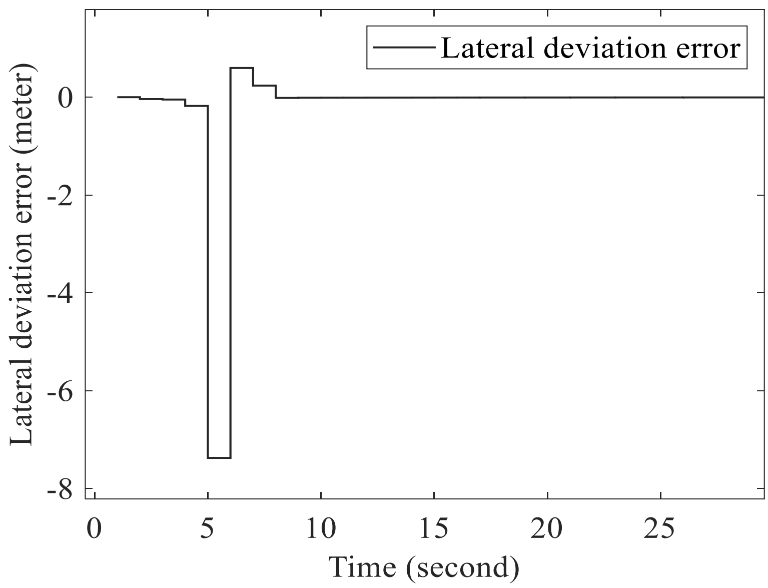

Figure 6. Lateral deviation error.

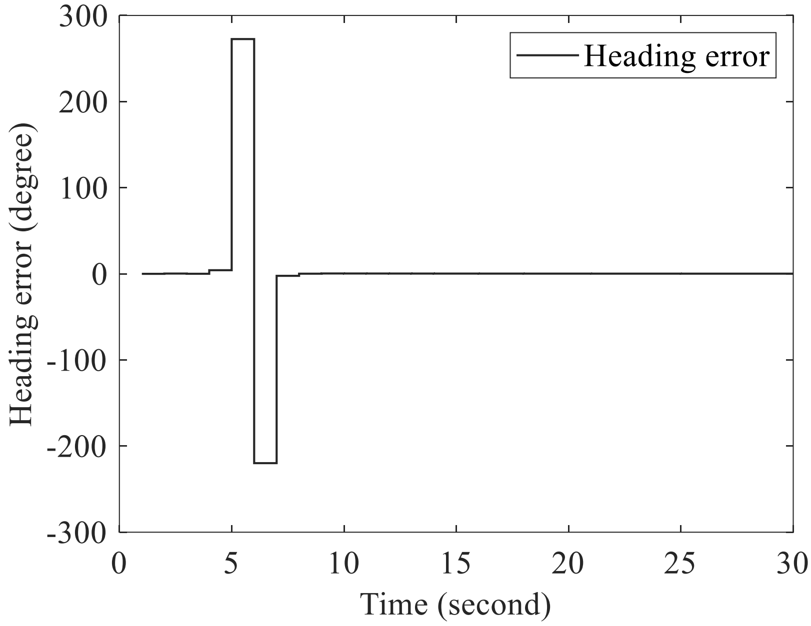

Figure 7. Heading error.

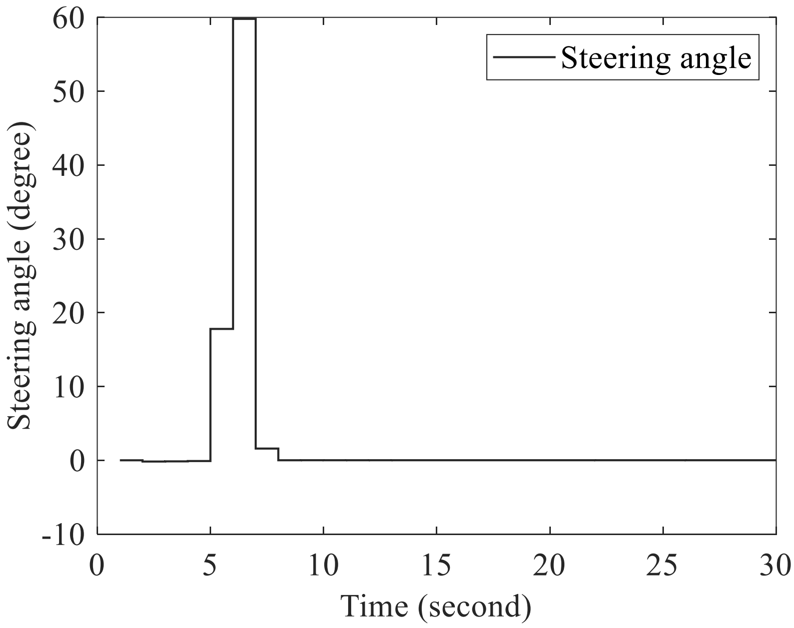

Figure 8. Steering angle.

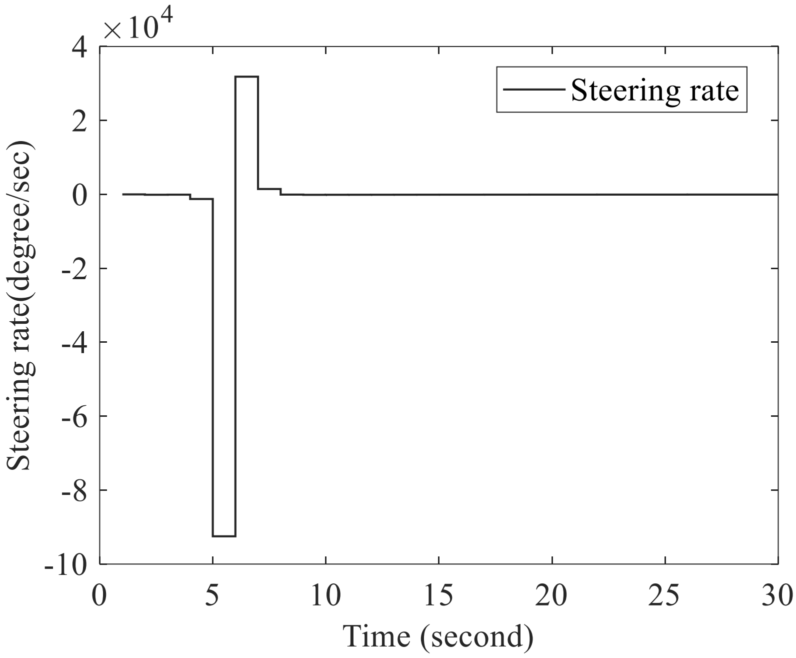

Figure 9. Steering rate

Figure 4 presents the sideslip angle convergence to equilibrium point in 10 seconds. Figure 5 shows that the yaw rate converges in 12 seconds. Figures 6-9 show that the lateral deviation error, the heading error and the steering angle converges in 8 seconds. These results show the effectiveness of our control design.

5. CONCLUSIONS

In this paper, we propose a control design for an autonomous vehicle. The discrete-time T-S system represents the autonomous vehicle model. The state vector is computed by using the Luenberger observer. Furthermore, we applied a PDC control law to the T-S fuzzy model. Feasible LMI conditions have been developed to guarantee lane keeping under certain limits. The results show that the lateral control of the autonomous vehicle to keep it on the centreline of the lane is well done under various constraints and scenarios. In future work, we will improve the heading error percentage to ensure more safety when we want to go back to the centreline of the lane.

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