# OPTIMAL TRAJECTORY OF ROBOT MANIPULATOR FOR ENERGY MINIMIZATION WITH QUARTIC POLYNOMIAL EQUATION FOR LINK-JOINT ANGLE 

Luping Luo, ${ }^{1}$ Kyoosik Shin ${ }^{2}$ and Changsoo Han ${ }^{3}$<br>Department of Mechanical Engineering, Hanyang University, Ansan City, Korea<br>${ }^{1}$ luping860317@gmail.com<br>${ }^{2}$ norwalk87@hanyang.ac.kr<br>${ }^{3}$ cshan@hanyang.ac.kr


#### Abstract

In this paper, a different way to find the trajectory of the robot manipulators for energy optimization is presented. In our method, the joint angles of the manipulator are set as quadratic polynomial functions. Then, with them taken into the variational function of energy consumption, Finite Element Modelling is employed to optimize the unknown parameters of the fourth order joint angles.


## KEYWORDS

Trajectory, Manipulator, Energy Optimization, Quartic Polynomial equation, Finite Element Modelling

## 1. INTRODUCTION

Nowadays, robot manipulators have been applied broadly into almost all the areas. The industrial robot manipulators (IRM) have been commonly used, and also high technology is getting used to improve the performance of them [1] [2]. The auxiliary robot manipulators (ARM) have been started taking into application, such as wearable robot manipulator X-Ar. In such situations, three or more arm manipulators are commonly used. They are in great demand to speed up the automation processes [3] [4]. For all the robot manipulators, electricity and battery are the mainly ways for energy supply.

Nowadays, the energy saving has been crucially important due to environment problems and energy crisis. The energy saving technology has not been established in robotics industry. Many methods have been presented for energy saving. Such as, control techniques for minimize energy consumption [5] [6], assist robot manipulator [7] [8]. The GA can be used to search the parameters of the polynomial to minimize the energy consumption. Intelligent methods like fuzzy logic, neural network, genetic algorithm etc. are widely used in different areas, especially in advanced computing, control and optimization problems. Intelligent methods have already been used in optimization of movement and trajectory planning of manipulators [9] [10]. But the problem is the calculation is too complex and hard for application. So we present to set the trajectory as polynomial which makes the calculation much easier.

In this paper, the joint angles of the robot manipulator are expressed as quadratic polynomial function. First, we set all the joint of the robot manipulator as fourth order polynomial function.

Here, all the joints of the robot manipulator are revolute joints. Then, we search the relationship between the electric energy and the joint torque required to finish the task for the robot manipulator. And then, get the fourth order polynomial function of joint angles taken into the electric energy consumption function for optimization. At last, we take two-link robot manipulator as the model for the simulation to check our method.

## 2. Formulation for Energy Consumption

By setting the joint angles of the robot manipulators as quadratic polynomial equations, the energy consumption of the robot manipulators can be gotten.

### 2.1. Quadratic Polynomial Function for Joint Angles

When the trajectory reference is calculated with considerable precision, a large amount of data is required. Thus, to make the feasible manipulation of data, some function to represent the reference data is required. In this paper, the fourth order polynomial function is selected to represent the joint angle.

The j-th joint angle of the robot manipulator is set as the quadratic polynomial function by

$$
\begin{equation*}
q_{j}=\mathrm{a}_{j 0}+\mathrm{a}_{j 1} \mathrm{t}+\mathrm{a}_{j 2} \mathrm{t}^{2}+\mathrm{a}_{j 3} \mathrm{t}^{3}+\mathrm{a}_{j 4} \mathrm{t}^{4} \tag{1}
\end{equation*}
$$

Where a0, a1, a2, a3, a4 are all scalars.
The boundary conditions are given by


With (2) taking into (1), $a_{j 0}, a_{j 1}, a_{j 2}, a_{j 3}$ are gotten. Then there just $a_{j 4}$ are the unknowns. When time $t$ and scalar $a_{j 4}$ are given, the joint angle, joint angle velocity and joint angle acceleration can be gotten.

### 3.2. Energy Consumption Equation

When a DC motor driver is used in each joint, the total electrical energy consumed [11] from the robot manipulator is calculated as follows:

$$
\begin{equation*}
E_{t}={\underset{j}{\circ}}_{j=1}^{m} \grave{\mathrm{O}}_{t_{0}}^{t_{f}} u_{j}{\grave{x_{a}}}_{a_{j}} d t \tag{3}
\end{equation*}
$$

The electrical equations of DC motor drivers at the $j$-th axis are written as follow:

$$
\begin{align*}
& u_{j}=R_{a_{j}} i_{a_{j}}+L_{a_{j}} \frac{d}{d t} i_{a_{j}}+E_{a_{j}}  \tag{4}\\
& T_{j}=K_{t_{j}} i_{a_{j}}  \tag{5}\\
& E_{a_{j}}=K_{e_{j}} \frac{d}{d t} q_{a_{j}} \tag{6}
\end{align*}
$$

Where: $R_{a}, K_{e}$, and $K_{t}$ are a resistance, EMF constant, and torque constant of the motor. $u, i$, $E_{a}, T$ are the voltage, current, EMF and torque of the motor, respectively.

The general dynamic equation of the robot manipulator is

$$
\begin{equation*}
M(q)+V(q, q+G(q)=t \tag{7}
\end{equation*}
$$

Putting equation (1) through (7) to (3), and grouping the terms yields the following equation concerning the total electrical energy:

$$
\begin{align*}
& +d(q) e(q) q \not q(q)] d t \tag{8}
\end{align*}
$$

where:

$$
\begin{aligned}
& A(q)=R_{a} \times K \times M(q)^{T} \times M(q) \\
& B(q)=R_{a} \times K \searrow \forall(q) \times M(q)+K_{e} \times K t^{1} \times M(q) \\
& C(q)=R_{a} \times K \rtimes \forall(q)^{T}+K_{e} \times K t^{1} \ngtr V(q) \\
& d(q)=R_{a} \times K \searrow \backslash(q) \times G(q) \\
& f(q)=G(q)^{T} \times R_{a} \times K \nsucc G(q) \\
& K=\left(K_{t} \times K_{t}^{T}\right)^{-1} \\
& R_{a}=\operatorname{diag}\left(R_{a l}, R_{a 2}, \mathrm{~K}, R_{a n}\right) \\
& K_{t}=\operatorname{diag}\left(K_{t 1}, K_{t 2}, \text { K , } K_{t n}\right) \\
& K_{a}=\operatorname{diag}\left(K_{e l}, K_{c 2}, \mathrm{~K}, K_{e n}\right) \\
& n=\text { number of axis }
\end{aligned}
$$

## 3. Simulation

This section shows the simulation results by using the quadratic polynomial equations for joint angles of robot manipulators.

### 3.1. Simulation Model

For the simulation, we take the two-link manipulator as shown in Figure 1 as our model. We just take the revolute joints into consideration. The link masses are all set on the joints of each link where the motors are also set on.


Figure 1. Two-link manipulator for research.

### 3.2. Simulation Conditions

The cycle $T$ was set as 1 s . The physical parameters of multi-joint robots were given in Table1. The conditions were set as


Table 1. Physical parameter of simulation model

| Parameters[unit] | Link 1 | Link 2 |
| :---: | :---: | :---: |
| Length $[\mathrm{m}]$ | 1 | 1 |
| Mass $[\mathrm{kg}]$ | 1 | 1 |
| Inertia moment $[\mathrm{kg} . \mathrm{m} 2]$ | 1 | 1 |
| Length from mass centre to joint $[\mathrm{m}]$ | 0.5 | 0.5 |

### 3.3. Simulation Algorithm

For simulation, the energy consumption equation can be as

$$
\begin{equation*}
E=f \text { égeg }(t), t, a_{j 4} \text { ù } \tag{10}
\end{equation*}
$$

Performance Index is set as

$$
\begin{equation*}
J=f \text { 色 }\left(t_{\text {opt }}\right), t_{\text {opt }}, a_{j 4 \text { opt }} \hat{\mathbf{u}} \tag{11}
\end{equation*}
$$

Subject to

$$
\begin{equation*}
j \underset{e^{2}}{g}\left(t_{\text {opt }}\right) \text { 㐫 }=0 \tag{12}
\end{equation*}
$$

### 3.3.1 Main Program

1. Choose an initial guess of $t_{\text {opt }}$ and $a_{j 40 p t}$
2. Solve $E(t)=f[q(t), t]$.
3. $\operatorname{Find} f\left(q\left(t_{\text {opt }}\right), t_{\text {opt }}, a_{\text {joopt }}\right)$ and $j\left(q\left(t_{\text {opt }}\right)\right)$.
4. Update $t_{\text {opt }}$ and $a_{\text {j4opt }} \quad$ such $\quad$ a way to minimize $f\left(q\left(t_{\text {opt }}\right), t_{\text {opt }}\right) \quad$ while maintaining $j\left(q\left(t_{\text {opt }}\right)\right)=0$.
5. Repeat 2 to 4 until termination conditions are satisfied.

### 3.3.2 Subprogram

1. $\quad \operatorname{Set} t=t_{\text {max }}, q\left(t_{0}\right)=q_{0}$.
2. Solve $E(t)=f[q(t), t(t), t]$.
3. Find the first $t_{1}$ such that $\sin \left(q_{1}\left(t_{1}\right)\right)-1=0, \cos \left(q_{3}\left(t_{1}\right)\right)-1=0$ or $q_{2}\left(t_{1}\right)=0$, $q_{4}\left(t_{1}\right)=0$.
4. If the former case, terminate, otherwise store u and $t_{1}$.
5. $\quad$ Set $t_{0}=t_{1}$. Go to 2 .

### 3.4. Simulation Results

With Subprograms we can get the initial angle and angle velocity of link 1 and link 2. Then taking the initial results into the main programs, the optimal angle and angle velocity of link 1 and link 2 can be got. Figure 2 and Figure 3 shows both the simulation result of link 1 and link 2. Figure 4 shows the torque consumption of link 1 and link 2 of the two-link manipulator. Figure 5 shows the optimal trajectory of the two-link manipulator.


Figure 2. Initial and optimal Angle and angle velocity of link 1.


Figure 3. Initial and optimal Angle and angle velocity of link 2.


Figure 4. Torque consumption of link 1 and link 2.


Figure 5. Optimal trajectory of two-link manipulator.

## 4. CONCLUSIONS AND FUTURE Plan

In this paper, quadratic polynomial equation is used for the joint angle of the robot manipulator. Then, the joint angle velocity and joint angle acceleration of the robot manipulator can be gotten. With the gotten joint angle, joint angle velocity and joint angle acceleration of the robot manipulator taken into the energy consumption equation, we can get the optimal trajectory for energy minimization.

In previous research, the torque given by the motor is large enough to drive the robot manipulator. However, in some condition, the torque is not big enough to arrive the asked arrive position. In the future, we will do the research on torque limitation of robot manipulator.

## Acknowledgements

"This research was supported by the MKE(The Ministry of Knowledge Economy), Korea, under the 'Advanced Robot Manipulation Research Center' support program supervised by the NIPA(National IT Industry Promotion Agency)" (NIPA-2012-H1502-12-1002)

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